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College of Education
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mshores@uab.edu

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903-886-5538
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The Importance of Advocacy for Integration and Effective Instruction

The world around us is integrated—as we use our knowledge of science, mathematics, technology, engineering, language arts, social studies, the arts, and skills of problem solving, critical thinking, and communication to navigate our communities on a regular basis. Why is the way we teach our children not aligned with our daily experiences? As I have worked this past year on standing up and implementing a state-level GEAR UP (Gaining Early Awareness and Readiness for Undergraduate Programs) program it has become increasingly clear to my team and me that classrooms across our state are using direct instruction as the primary means to teach students disciplines including science. The teaching of mathematics has been reduced to a series of locksteps on a fill-in-the-blank worksheet to be followed to get the answer. A wise teacher I once worked with in the southwest told me, "I don't think having students parroting back information translates into learning anything." The literature base is clear and it supports this teacher's statement. Memorization of knowledge at a factual level does not promote student development of deep conceptual understanding (e.g., Johnson, Bolshakova, & Waldron, 2014). Our project is in 15 schools with high socioeconomic need spread across our state in urban and rural areas. Unfortunately, our observations are reflective of the reality of schools across our nation today. Many schools do not have access to high-quality, engaging STEM curriculum.

The research base is clear on the demonstrated impact of the use of pedagogies, including problem- and project-based integrated learning (e.g., Czerniak & Johnson, 2014) and culturally relevant teaching (e.g. Johnson & Fargo, 2014; Lee & Buxton, 2013) on student learning and achievement in STEM. Faculty should be sharing with school partners the research on best-practice strategies, and approaches for students from underrepresented backgrounds is needed to help grow student access to effective instructional experiences. Additionally, STEM educators should partner with K–12 schools and advocate for change, beginning with teacher professional development, curriculum development support, and other resources to give schools an option to traditional modes of teaching the disciplines. Schools will need resources to make the transformation from direct and traditional instruction to a more student-centered approach. STEM educators should engage with their local legislators and policy makers to advocate for funding to schools to help provide the resources necessary to support implementation of new research-based approaches. It will take a village, a large one, to enact and make the change that is needed in many of our schools.

Carla C. Johnson
Purdue University

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Preservice Secondary Mathematics Teachers' Conceptualizations of Equity: Access and Power as Seen Through Vignette Responses

Brooke Max
Purdue University

Attention to equity in the mathematics education field has been growing in recent years. We have evidence that many novice secondary mathematics teachers do not feel prepared to teach in regards to diverse populations. We need to know more about how secondary preservice mathematics teachers (PSMTs) conceptualize equitable environments. This study investigates 30 secondary PSMTs' proposed responses to two hypothetical vignettes from mathematics department conversations regarding calculator usage and mathematical discourse, respectively, utilizing two of Gutiérrez's four dimensions of equity: Access and Power. Results suggest these PSMTs considered equity, equality, and creating a classroom that invites participation among other factors when thinking of an equitable approach with respect to calculator usage. When considering mathematical discourse, PSMTs cited the need to "model" proper use of mathematical language as well as to allow students to themselves verbalize it. Implications mathematics education and teacher education more broadly are to integrate equity and equality discussions in methods courses and to include strategies to facilitate productive discourse.

Attention to equity in the field of mathematics education has been growing across the last few decades, and Secada, Fennema, and Byrd (1995) proposed the expansion of areas of equity-based inquiry that anticipate new social, research, and policy areas. Equity is operationalized as "a check on the justice of specific actions that are carried out within the educational arena and the arrangements that result from those actions" (Secada, 1989, p. 69). More recently, Gutiérrez (2012) called for more research on "effective/ successful teaching and learning environments" (p. 32) for marginalized students. Furthermore, equity is the first of six principles included in the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics* (*PSSM*) (NCTM, 2000). This principle calls for teachers to set and maintain "high expectations and strong support for all students" (p. 12), embracing and incorporating their diverse backgrounds into the mathematics classroom. However, the manner in which teachers are prepared to implement these recommendations requires sustained investigation as our nation continues to evolve (e.g., in demographics, rise of technology).

Many novice secondary mathematics teachers do not feel prepared to teach diverse populations, "including culturally diverse children, children living in poverty, and children who are otherwise at risk" (Desimone et al., 2013, p. 63). One possible reason for this is that teacher education programs generally separate the teaching of diversity issues from other parts of teacher education (Hollins & Guzman, 2005). We also know preservice secondary teachers can take the majority of their content coursework separate from their teacher education work. Teacher education programs need to move away from general skill and knowledge development into encouraging connections of content and context with preservice teachers (Darling-Hammond, Hammerness, Grossman, Rust, & Shulman, 2005). Conceptualizing equity specifically in mathematics education (e.g., access to mathematical resources, opportunity to engage in mathematical discourse, ability to enroll in upper-level mathematics courses) (Gutiérrez, 2012) could aid preservice secondary mathematics teachers (PSMTs) in supporting students once they are in their own classroom and is necessary if we want teachers to embrace the full spirit of the equity principle.

For some, equity may mean equality of treatment, where for many in the mathematics education community, equity follows the NCTM *Principles to Actions'* (2014) description of access and support "to maximize learning potential" (p. 59) through high-quality curriculum and teachers along with high expectations. Strategies that can support equity-based mathematics teaching allow PSMTs to reflect on their own practices and beliefs, to build on students' knowledge, and to create a mathematics community in the classroom that values students' strategies (Chao, Murray, & Gutiérrez, 2014).

However, due to their own upbringing and understanding of the education system, PSMTs may not be aware of the fact that traditional school practices can clash with the home cultures of students from diverse backgrounds (e.g., Delpit, 1995; Paley, 1979). We can help students empower themselves through using equity-based practices but also by communicating the rules of the culture of power, a middle-class White man's world, which include "linguistic forms,

communicative strategies, and presentation of self; ways of talking, ways of writing, ways of dressing, and ways of interacting" (Delpit, 1995, p. 25). To help PSMTs learn to create and support equitable environments as defined by NCTM, we first must better understand how they conceptualize equity.

For this study, I begin with a review of literature related to equity in education and more specifically in mathematics education programs, paying close attention to areas of *access*, *achievement*, *identity*, and *power*, as dimensions of equity highlighted by Gutiérrez (2009). I then discuss PSMTs' conceptualizations of equity through their responses to vignettes involving hypothetical discussions with other mathematics teachers at their "future school." Finally, I discuss the themes that emerged from these PSMTs' responses as well as implications for this work in secondary methods courses and possible future research.

Literature Review

Equity in Education

Equity considerations are not new to the educational field. Bringing light to how teachers conceptualize their role in supporting equitable environments in their classroom, Paley (1979) discussed internal battles she experienced when teaching students of diverse backgrounds, particularly the cultural differences in the Gentile/Jew relationship she experienced as a child compared to the Black/White dynamic she saw in her own classroom. Delpit (1995) spoke of this cultural conflict in terms of power in the classroom and how learning the rules of a culture of power makes it easier for those not familiar with the culture to acquire power (Delpit, 1995; Gutiérrez, 2009). PSMTs need to be cognizant of these power tensions that exist as they think about the diverse populations in which they will teach and how to cultivate equitable environments.

Specifically in teacher education, Hollins and Guzman (2005) found 101 studies between 1980 and 2002 regarding preparing teachers to teach diverse populations in the United States. They found that candidates generally lacked confidence to do well in diverse settings and preferred not to be placed in situations where they felt uncomfortable. Another key finding in this study was that often teacher education programs separate diversity from content-specific pedagogy. Not only are the diversity courses separated, but these courses, generally in multicultural education, begin to challenge PSTs' beliefs about diverse populations, not necessarily help them learn how to teach diverse populations (Gomez, 1996). In a review of 80 studies of effects of preservice teacher education strategies for preparing teachers for culturally diverse schools, Sleeter (2001) found the focus appeared to be on attitudes and lack of knowledge of White preservice teachers. While this perspective is important, we also must consider what PSMTs believe is an equitable environment and find strategies to support them in their emerging practices. These studies highlight the need for us to consider strategies that can be employed in teacher education to aid them in creating equitable environments.

Equity in Mathematics Education

Issues of equity in mathematics education have been framed in many ways. One is toward "gap-gazing" (Gutiérrez, 2008, 2012, p. 31), and another is "gaps analyses" (Lubienski, 2008). These theories have mutual interests of finding ways to best support all mathematics students, as does the work with preparing preservice elementary teachers to enter culturally diverse classrooms (e.g., Downey & Cobbs, 2007; Turner et al., 2012), particularly increasing PSTs' awareness of the diversity in the classroom. However, more needs to be known about the ways in which PSMTs conceptualize equity in the classroom.

Secada (1989) discussed how equity and equality can often be hazy in mathematics education. One succinct way to think about issues of equity arises from Gutiérrez's (2012) definition of equity as "fairness, not sameness" (p. 18). Specifically in equity, Gutiérrez proposes four dimensions and their characteristics: *access*, *achievement*, *identity*, and *power*. These four dimensions are categorized into two axes—the *dominant* axis contains access and achievement, the areas of equity most focused upon in the 1980s and 1990s (Gutiérrez, 2012) and where groups that attempt to close the achievement gap (e.g., Teach For America) give much attention. Access refers to the physical, tangible resources made available to students (e.g., "high-quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom environment that invites participation, reasonable class sizes, and supports for learning outside of class hours") (Gutiérrez, 2012, p. 19). Achievement refers to student outcomes, particularly with measurable results, particularly "participation in a given class, course-taking patterns, standardized test scores and participation in the math 'pipeline'" (p. 19).

Identity and power are contained in what Gutiérrez deems the *critical axis*, where students' cultural backgrounds are valued and mathematics can be viewed as a tool to effect change. Specifically, identity refers to students' ability to see the world through mathematics and find themselves in the mathematics in ways such as: using mathematics to analyze social justice issues, finding

mathematics useful to their lives, opportunities to use cultural and linguistic resources. Power refers to social transformations in the classroom that include: "voice in the classroom, opportunities for students to use math as an analytical tool to critique society, alternative notions of knowledge, rethinking the field of mathematics as a more humanistic enterprise" (p. 20). Gutiérrez (2012) is clear that all dimensions of equity will not be represented equally in any given situation. In fact, the dimensions are interrelated and not isolated. However, in this study, due to the design, I will explore one dimension of the dominant and critical axes: access and power, respectively.

Access and power. First, I will discuss calculators in the classroom in terms of availability and results from this access. When implemented effectively, calculators can be a useful resource in the classroom, as Farrell (1990) reported more activity in the classroom among students using graphing calculators. In fact, a meta-analysis of 54 studies of the effects of calculators on students' achievement and attitude levels found that pre-college students who merely had access to calculators had better attitudes toward mathematics than those who did not, even though the mathematical development was not affected (Ellington, 2003). More recently, Olive et al. (2010) found:

> ...interactions among students, teachers, tasks, and technologies can bring about a shift in empowerment from teacher or external authority to the students as generators of mathematical knowledge and practices; and that feedback provided through the use of different technologies can contribute to students' learning.

These findings not only have implications for access, but also power when considering interactions in the classroom and students as generators of knowledge.

Similar to Delpit (1995) talking about power in the classroom, Boaler and Greeno (2000) discussed power specific to the mathematics classroom being held in language and register. The constructivist view that "learning is a process by which students generate meaning in response to new ideas and experiences" (Villegas & Lucas, 2002, p. 25), and knowledge is not inert information passively transmitted from teachers to students is critical for PSMTs to understand. There is a dynamic aspect to learning that is constructed through discourse in the classroom, and there are various ways in which discourse can be facilitated with students: spontaneous helping, peer teaching, reciprocal critique, collaborative problem solving, and through social relationships among students (Cazden & Beck, 2003). Other ways to support productive mathematical discourse are: valuing students' ideas, exploring students' answers, incorporating students' background knowledge, and encouraging student-to-student communication (White, 2003). These can come through revoicing students' words, asking students to restate someone else's reasoning, and using wait time (Chapin, O'Connor, & Anderson, 2003). How these strategies are employed can have implications in regard to who holds power in the classroom (e.g., which students are actually doing the talking, which students are encouraged to teach others).

However, we must still know more about how PSMTs conceptualize their role and ability to create equitable environments for their future students. Specifically, I ask: How do PSMTs plan to respond in equity conversations with mathematics teachers involving calculator usage and mathematical discourse in the classroom? What considerations do PSMTs make when conceptualizing equitable environments?

Methods

Setting and Context of the Study

This study was conducted at a large Midwestern university in a secondary mathematics methods course. Students in this program receive a Bachelor of Science in Mathematics Education. The course, in the College of Education, traditionally is taken in the semester prior to student teaching and completion of the program. As part of the course, students participated in 15 hours of field experience, mostly in a secondary mathematics classroom, wrote a research paper investigating a classroom technique of interest to them (e.g., argumentation in the classroom, implementing technology in the classroom), and created student-centered lesson plans designed to reach all learners and successfully integrate technologies. Several topics in the course were: the Common Core State Standards for Mathematical Practice, *PSSM* content areas (Algebra, Geometry, Measurement, Number & Operations, and Data Analysis & Probability), Equity, and Classroom Management.

Participants

The participants were 30 undergraduate and master students enrolled in two fall terms of a secondary mathematics methods course. All students in both terms responded. Eighteen of the students were male, which was reflective of the university's demographics. Twenty-six students were White, two were Black, one Latino, and one Asian. Eight participants were students obtaining their teaching license through a master's program in mathematics education. The remaining 22 participants were undergraduate students with a few unique situations. For

example, one was a junior in the program, taking it a year prior to the other traditional students, one was a technology education student who wanted to expand his mathematics background, and one was an international student hoping to teach in the United States. Multiple students in the course had already been accepted to Teach For America, where other students had voiced preferences to teach in a private religious school upon graduation.

Role of the Researcher

The researcher earned a Bachelor of Science in mathematics education and taught secondary mathematics for 11 years that included two years in an inner-city through a Teach For America and nine years in the rural Midwest. She earned a Master of Education in curriculum and instruction and was a co-instructor of the secondary mathematics methods course during both semesters of the study. Three other instructors co-taught the course during the implementation of the study, two with university mathematics teaching experience and one with secondary mathematics teaching experience.

Data Sources and Materials

A group of researchers, including one mathematics education professor and four mathematics education doctoral students, created five vignettes to present to the PSMTs for an intended program self-evaluation. Each vignette was designed to address one of the following areas: equity, discourse, problem solving, modeling, and assessment in relation to mathematics education.

Students viewed the vignettes digitally, and each exchange was presented on a different screen. To provide contextual information, each vignette started with hypothetical school background information presented as a running head, followed by the hypothetical discussions from weekly departmental meetings. To allow the participants to consider a realistic future school, specific school demographics were not included (e.g., ethnic make-up, socioeconomic information). Students typed and submitted their responses into a familiar online forum that they used repeatedly throughout the semester. Students had one week from the in-class announcement to respond and were asked to do so on their own time, outside of the class. Because all responses were open-ended, there was to be an expected variety in student responses.

The background information as well as exchanges one and two are as follows, with equity and discourse, respectively:

> **Background.** The mathematics teachers at Sunny Cliff High School have weekly departmental meetings to discuss issues related to their teaching. Below are pieces of their conversations on which to reflect. For each exchange, give your thoughts, contributing to the discussion as if you were a teacher in the department. This assignment will not be graded, but counts as class participation.
>
> ***Exchange 1.***
> Ms. Lopez: I encourage some, but not all, of my students to use calculators in class. If I don't let these students use calculators, they can't contribute to the problem solving we're doing.
> Mr. Parker: I too have students who benefit from the use of calculators, but I think fairness is really important. Depending on the lesson, I either let all of my students use calculators or none of my students. This way no one ever feels cheated.
>
> ***Exchange 2.***
> Ms. Booth: Because I know more mathematics than my students and they look to me as the expert, I do most of the talking. It's important for students to hear the correct uses of mathematical language so I model that as much as possible.
> Ms. Sutherby: Students will learn mathematics by using the language themselves, even if imperfectly, so I let them talk as much as possible.

Data Analysis

Each response from the PSMTs was first coded by exchange, in regards to whether their responses encouraged one teacher's approach solely, a balance of the two, or it was unclear. In cases where the preservice teachers said explicitly "I agree with...," those were coded into the category of the teacher listed. If it was not explicit, context was considered to determine with which category the comment aligned. For example, "I give the students the option of using a calculator. If the student does not want to use one, then the student does not have to use it" was classified as aligning with a mixture of the two because the PSMT was giving access to all of the students and allowing for individual student needs to determine use.

After this stage of the analysis, each response was coded with the Gutiérrez (2009) framing of equity in the four dimensions of access, achievement, power, and identity in mind. An example of coding in each category can be seen in Table 1.

Analytical coding (Richards, 2015) was implemented to find themes within each exchange as well as looking at the four dimensions across all responses. Based on the large number of references to access in the calculator usage vignette and power in the mathematical discourse vignette, themes were explored in-depth in those areas only. The

Table 1
Coding Scheme

Aspect of Equity	Description (Gutiérrez, 2009)	Example
Access	Resources available (e.g., adequate technology and supplies, classroom environment that invites participation)	"I think it is important to create a fair situation, so all students should have access to a calculator. However, the word equity comes into play because some students will need additional resources. While calculators are a good thing to create overarching rules for, other things can be adjusted based on the needs of the students."
Achievement	Student outcomes (e.g., participation in a given class, standardized test scores)	"I think that Mr. Parker has an excellent point. Either all students should be allowed the use of a calculator or not unless the student has a specific learning disability, etc. I think that many students would feel that it is unfair for some students to use calculators and some to not be allowed. For the students who can't contribute without the use of a calculator, they need to receive extra help in order to catch them up to the level of the other students."
Identity	Balance between self and others (e.g., gendering and classing of students, opportunities to draw upon cultural and linguistic resources)	None available
Power	Issues of social transformation (e.g., who gets to talk in the classroom, opportunities to use math as an analytic tool to critique society)	"I think it is important for the teacher to always model mathematical language; however, I think it is more important that teachers get their students to start talking about math as much as possible in the classroom. The more they talk about math, the more comfortable they will feel about using mathematical language."

responses were read and reread, noting statements that seemed revealing in regard to Gutiérrez's dimensions of equity and coded to multiple themes if more than one theme was expressed. A second person coded 30% of the responses with 85% compatibility.

Six themes emerged from the calculator usage vignette. The codes along with one example of each can be found in Table 2. In the mathematical discourse, five themes emerged (see Table 3).

Findings

Calculator Vignette

The responses for the calculator usage vignette were first analyzed according to which approach the PSMTs encouraged. The approach framing fairness in terms of equality was supported by 53% of the PSMTs ($n = 30$), 17% of the PSMTs reacted with supporting the encouragement of some students, and 30% of the PSMTs reasoned that a balance of the two approaches should be sought. The equality responses included: "I agree with Mr. Parker; I think that if I'm going to use calculators it should be an option for all the students."

Encouraging some students included responses such as: "Ms. Lopez, I agree with you that there are some students that I encourage to use calculators in class. Some students who struggle with arithmetic are not able to effectively participate in class if they do not have one because they make too many mistakes or get caught up with doing the arithmetic very carefully. Having a calculator allows them to focus on understanding the new material and new concepts, instead of getting the arithmetic of the problem correct."

The balance of equality and equity categories included responses such as: "I think if I'm going to let some students use calculators I should give the option to everyone. However, I might encourage students who feel more advance[d] to try to do problems without their calculators." Table 4 summarizes the frequencies of the aforementioned themes of calculator usage according to the access aspect of Gutiérrez (2009). The results in Table 4 show that equality was the most common theme for these PSMTs with the thought of the calculators as a resource for the students also appearing in half of the responses. One student specifically seemed to think about fairness in terms of equality and equity when saying:

> I think it's really important for each student to feel that they are being held to the same standard as all the other students. For that reason, I would hesitate from allowing different usage of tools in the classroom

Table 2
Coding Themes for Calculator Usage Vignette

Theme	Example
Calculators as a resource for the students	"Students should have their resources in the real world and they have gone past arithmetic."
Fairness (in terms of equity)	"The students who need more help can use the calculator and the advanced students who don't need it, don't need it."
Classroom that invites participation	"Some students who struggle with arithmetic are not able to effectively participate in class if they do not have one because they make too many mistakes or get caught up with doing the arithmetic very carefully."
Equality	"Either all students should be allowed the use of a calculator or not unless the student has a specific learning disability, etc. I think that many students would feel that it is unfair for some students to use calculators and some to not be allowed."
Lesson needs	"If the lesson is not focused on those reasons there really is no reason to keep students from using a useful tool, especially if others are allowed to use them."
To avoid social consequences	"The struggling students have enough academic burdens as it is; I don't want to potentially add any social burdens on top of that."

> depending on students' abilities. I would, though, individually challenge students who I know don't need to use their calculator to solve a problem to do so without it.

Of the 11 students mentioning fairness as equity, five of them also explicitly mentioned equality in their response, showing the desire to find a way to incorporate both. Another student commented:

> I think that if I'm going to use calculators it should be an option for all the students. I wouldn't want anyone to feel singled out or excluded. I understand that some students may need them more than others, but by making it an option all those kids that need them can pull them out without feeling inferior to those that don't need them because everyone had the chance to use one if they wanted to. It changes it from appearing like a necessity—"I can't contribute without using a calculator"— to a desire—"I want to use a calculator to contribute.

This particular response was an example of one response coded into four themes: equality because the option was to be given to all students, fairness as equity due to the fact that the PSMT recognizes not all students need them in the same way, classroom that invites participation because of the desire to allow all students to be able to participate, and to avoid social consequences for students due to the mention of "feeling inferior." In part of the response, one student referenced: "Equity allows for students to have different adjustments made for them so that they can all have an opportunity to learn." This specific line was classified into the fairness as equity category and supports the NCTM view of equity.

Discourse Vignette

The mathematical discourse vignette responses were also analyzed according to which approach the PSMTs encouraged. The approach of having the students talk as much as possible was supported by 30% of the PSMTs ($n = 30$), 7% of the PSMTs encouraged a teacher-majority approach at discourse, and 63% of the PSMTs reasoned that a balance of the two approaches should be sought. An

Table 3
Coding Themes for Mathematical Discourse Vignette

Code	Example
Student practice talking mathematically	"It is important to have them practice using mathematical language when discussing mathematics with other students."
Teacher modeling	"It is important to model correct mathematical language."
Value in making mistakes	"We do need to correct them when they are wrong, but there is something to be said about failure and learning."
Deeper learning	"Allowing the students to do most of the talking gives them the opportunity to create the structures and solutions on their own. This provides a deeper understanding of the material."
Ability to communicate mathematically	"They feel more comfortable and are able to communicate better mathematically."

Table 4
PSMT Responses Regarding Access and Calculator Usage

Theme	Percentage of PSMTs Referencing ($N = 30$) (%)
Equality	57
Calculators as a resource for the students	50
Fairness (in terms of equity)	37
Classroom that invites participation	23
To avoid social consequences	13
Lesson needs	10

Table 5
PSMT Responses Regarding Power and Mathematical Discourse

Theme	Percentage of PSMTs Referencing ($N = 30$) (%)
Student practice talking mathematically	93
Teacher modeling	70
Value in making mistakes	37
Deepen learning	37
Increased ability to communicate mathematically	10

example of a response categorized as the student majority approach included:

> I agree, it's important to let them do most of the talking. Even if we know more mathematics as teachers, students learn a lot more when they get to do the math in class because it lets us know where their misconceptions are and we can address that.

The teacher doing the majority of the talking was a very limited approach. One example of a response categorized in this manner was:

> Allowing students to talk as much as possible is just not something that is reasonable. If a teacher would let the students talk as much as possible, the lesson would never get completely covered. Sure, it is important for the students to talk through problems but it needs to have bound[a]ries.

The majority of the PSMTs found a balance of the teacher and student talking to be important, with one PSMT responding:

> It is important to find a balance when it comes to who does the talking in the classroom. While it is important to make sure that students are given an opportunity to hear the correct mathematical language, practicing is how they will come to truly understand math. Even if they make mistakes along the way, if students are given the opportunity to speak themselves their learning will be much more meaningful.

Table 5 summarizes the results of the same vignette coded analyzed through the lens of the power (Gutiérrez, 2009) aspect of equity. As seen in Table 5, PSMTs referenced the ability for students to practice mathematical discourse the most of any other power aspect, particularly with statements like, "It is important to have them practice using mathematical language when discussing mathematics with other students." Teacher modeling was also viewed as a priority, as seen in the 70% respondents. The value PSMTs saw in making mistakes was revealed with, "Even if they make mistakes along the way, if students are given the opportunity to speak themselves their learning will be much more meaningful." One statement that encompassed many of the themes was: "I think it is important for the teacher to always model mathematical language, however, I think it is more important that teachers get their students to start talking about math as much as possible in the classroom. The more they talk about math, the more comfortable they will feel about using mathematical language."

Discussion

The purpose of this study was to gain insights on PSMTs' conceptualizations of an equitable classroom through their responses to vignettes regarding calculator usage through an access lens and mathematical discourse through a power lens. The majority of the PSMTs in this study responded not only with an approach with which they would support in their own classroom but also with justification for their support, with many of them seeking a balance of the teachers' approaches. In written words, these PSMTs felt comfortable taking a stand for what they believed to be an equitable environment. These results have implications for teacher education programs and areas should be addressed in order to support the emerging equitable practices of PSMTs and preservice secondary teachers in general.

The responses to both vignettes indicated that these PSMTs were thinking about multiple factors as they envisioned their role in creating productive classroom environments. The responses to the calculator usage vignette indicated that more than half of these PSMTs discussed equality as an important part of an equitable

classroom, a notion that supports Secada's (1989) claim that equity and equality can be confusing in mathematics education. However, because 17% (n = 30) of the overall responses included equity as a part of equality, we see these PSMTs considering fostering an environment that supports student needs in ways such as these:

> I feel that fairness is important when dealing with students in math class. I feel that calculators can help those struggling to participate, and therefore I believe they should be allowed to use them. However, if there are students who would not benefit from the use of a calculator, I would try to inspire them to try problems without it.

A combination of these considerations could lead to a classroom environment where students feel valued with potential to open the door to classroom practices identified to support equity-based mathematics teaching as identified by Chao et al. (2014). Teacher education programs can design methods courses in such a way that equity is investigated as well as compared and contrasted with equality in relation specifically to the mathematics classroom and bring in these strategies to support preservice teachers in fostering those environments.

The emerging themes in the discourse example responses did not allow for Gutiérrez's (2009) power aspect to be used as fully as intended, but they did reveal that the PSMTs see the need for students to practice mathematical language. Moreover, these responses align with Cazden and Beck (2003) and Villegas and Lucas (2002) in the notion of knowledge not being passively received by the students. This suggests that they view student engagement in mathematical discourse as an important part in the learning process. The high percentage of PSMTs referencing student practice talking mathematically and teacher modeling of discourse, 93 and 70%, respectively, indicated that these PSMTs valued students participating in mathematical discourse. What it does not tell us, however, is whether the PSMTs feel comfortable and confident in their abilities to facilitate such discourse. It would be worthwhile to investigate the strategies of which PSMTs are aware (e.g., Cazden & Beck, 2003; Chapin et al., 2003; White, 2003) and feel comfortable implementing within the classroom to facilitate discourse. Teacher education programs can introduce and practice utilizing these strategies in methods courses of all content areas to support preservice teachers in implementing their conceptualizations of equitable environments.

One interesting characteristic of the responses is that no participant mentioned race, gender, or socio-economic status as a consideration in their responses. This could be a direct result of their backgrounds and inability to consider reconciling traditional school practices with the cultural backgrounds of their students, as referenced by Delpit (1995), or it could be due to the fact that the make-up of the school was not specified (e.g., demographics of the school or classes, resources available to the teachers or students) to allow for the freedom to visualize a future school where they would consider teaching. However, a future study that investigated equity in mathematics education would benefit from allowing participants to choose or describe a school where they would be teaching.

This study does show that PSMTs are thinking about equitable issues in their emerging practice in terms of access and power. Teacher education programs need to capitalize on this by integrating equity-based teaching strategies in methods courses and allow for opportunities for preservice teachers to think about equity in terms of equality and how they can structure their practices to best support the diverse populations they will inevitably encounter.

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Author's Note

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Using Culturally Responsive Stories in Mathematics: Responses From the Target Audience

Amy Corp
Texas A&M University Commerce

This study examined how Black students responded to the utilization of culturally responsive stories in their mathematics class. All students in the two classes participated in mathematics lessons that began with an African American story (culturally responsive to this population), followed by mathematical discussion and concluded with solving problems that correlated to the story. The researcher observed and recorded responses by students during each part of these lessons with protocols. Students independently reflected weekly by answering five questions to share their perspective on the African American stories. The teacher reflected on each lesson as well, describing thoughts on how these students responded to the story in each lesson. This paper examines the analyzed data from the target audience: Black students. Results revealed that Black students responded to the use of African American stories with high self-rated levels of engagement and enjoyment and that the stories helped them think about mathematics to varying degrees. Since students who are engaged and are thinking about mathematics are more likely to achieve mathematical understanding, the researcher concludes that this strategy should continue to be tested in diverse classrooms with an emphasis on student reflection to determine if the outcomes are transferable and generalizable.

Teaching Mathematics in the early grades is critical for students' understanding of how to use mathematics in everyday situations and is foundational for understanding mathematics in middle and high school. Students' behavioral and cognitive engagement is tied to achievement and is a predictor of later performance and even participation in risky behavior (Skinner, Furrer, Marchand, & Kindermann, 2008). Students who understand mathematics in the early grades are better prepared to enter higher mathematics courses required for college. This is our goal for all students. However, we understand this is not the reality for all students.

Engagement is a critical component of learning; without engagement students may become solely rote learners, missing essential mathematical connections. In mathematics, the connection between engagement and achievement is especially true for Black students in upper grades (Sciarra & Seirup, 2008). Black students who feel marginalized by their classmates, the teacher, or the curriculum often cope by becoming disengaged and withdrawn from learning (McMillian, 2003; Steele, 1992). McMillan (2003) gives the example of students looking like they are problem solving because they follow the procedural steps, such as underlining the question and circling the facts, but are cognitively disengaged because they did not connect to the problem and did not exhibit reasoning or critical thinking, often resulting in errors.

Lower standardized score for this population should not surprise us. In 2009 comparisons scores of the fourth-grade mathematics portion of the National Assessment of Education Program, Black students averaged a score of 222, which was 26 points less than White students (National Center for Education Statistics [NCES], 2011). Although there was a slight improvement from scores in 2009, only 66% of Black students were at the basic standard of achievement compared to 91% for White students (NCES, 2011). Noticeably, there was a significant gap between the performances of these two populations. Due to this reality and test scored described below, the unit of focus for this study is Black elementary students.

Skinner et al. (2008) conceptualize engagement in the classroom by looking at behavioral and emotional engagement and disaffection. The areas they describe for behavioral engagement are action initiation, effort, exertion, attempts, persistence intensity, attention, concentration, absorption, and involvement. The areas for emotional engagement are enthusiasm, interest, enjoyment, satisfaction, pride, vitality, and zest. Other reviewers of engagement they studied also looked at cognitive engagement (the students' mental interaction with the content). If mathematics lessons can engage these students behaviorally, emotionally, and cognitively, achievement should increase.

In recent research, some authors considered lower test scores of Black students as an achievement gap (Blank, 2011; Lee & Reeves, 2012), whereas others viewed these scores as an indicator of inequality (Gutiérrez, 2008; McMillian, 2003). Either way, the scores reflect that what schools have been doing has not been effective in

increasing the academic ability of Black students in mathematics as assessed by these standardized tests. In the participants' state, Black students earned the lowest percentage of students meeting expectations for mathematics in third grade in 2010 and 2011 (Texas Education Agency, 2012). So what does the research say about combatting disengagement for Black students in the mathematics classroom? What pedagogies have been successful in engaging Black students in school subjects?

This article examines how to engage students, particularly Black students, in mathematics at the elementary stage. The primary research question for this article is: How did Black students respond to culturally responsive stories in third-grade mathematics? Examining this capacious question captures actions and words that have the potential to describe how (behaviorally, emotionally, and/or cognitively) students were either engaged or disaffected in lessons build around stories with Black characters. The secondary question is: How did Black students engage with mathematics problems and activities tied to stories with Black characters? This broad question also allows for evidence of behavioral, emotional, and cognitive engagement by exploring all their responses to the mathematics, not just reactions or connections to the story (verbal, action, and written).

Plausible Interventions from the Literature Review

To engage students of a nondominant culture, I examined some culturally responsive practices. According to Gay (2002), "Culturally responsive teaching is defined as using the cultural characteristics, experiences, and perspectives of ethnically diverse students as conduits for teaching" (p. 106). Only one practice stood out as an ideal in the elementary setting: using stories featuring students' culture. The research literature reviewed indicates that using stories with strong Black main characters had positive academic effects on Black students within the language arts classroom and that students were more engaged when they saw their race represented in the content (Altieri, 1993; Hefflin, 2002; Hefflin & Barksdale-Ladd, 2001; Tatum, 2006). Thus utilizing Black literature as the context for teaching mathematics in the classroom was chosen in this study as the intervention or treatment for engaging Black students. There is very little research on teaching Black students mathematics at the elementary age. Ladson-Billings (1997) describes a positive teaching pedagogy with fifth graders of knowing the students' culture well and creating problems connecting to their daily life. She also mentions Smith and Stiff (1993), who created vignettes to connect to the students' interest (coming from their culture) for algebra in their urban classrooms. Ladson-Billings describes the research problem as such, "Certainly enough literature documents the mathematics failure of African American students. What is lacking is the documentation of successful practice of mathematics for African American students" (p. 706). Although there is little research on positive practices with Black students, the research for using stories in elementary mathematics classrooms has a broad base.

Teaching mathematics through stories has been confirmed as an effective pedagogy through research (Jennings, Jennings, Richey, & Dixon-Krauss, 1992; Keat & Wilburne, 2009; Hong, 1996; Tucker, Boggan, & Harper, 2010; Whitin & Whitin, 2004), and instruction for use as curriculum to connect students to mathematics in life activities (Moyer, 2000; Schiro, 1997; Shatzer, 2008). In these studies and other texts for promoting the use of literature, the majority of book titles suggested contain story characters that are either animals or White. Research into using multicultural books for teaching mathematics is nonexistent and is attributed to the relatively small amount of quality multicultural books available (Hefflin & Barksdale-Ladd, 2001).

This study extends the research of utilizing stories to teach mathematics by exploring the use of multicultural stories (specifically stories featuring Black characters). By observing students over the course of a semester during these lessons and analyzing the data from their own reflections, observations, and student work, I am able to provide a description of the qualitative outcomes. The overwhelmingly positive responses of students support using culturally responsive stories to engage Black students, even in culturally diverse classrooms because all students exhibited behavioral, emotional, and cognitive engagement. (The results of students by race will be given due diligence in a separate article.) Sciarra and Seirup (2008) found that behavioral engagement was the most significant indicator or learning and achievement for all students. Hopefully behaviorally engaging students in the doing and thinking of mathematics connected to a culturally relevant story primes students to learn and remember the foundational mathematical concepts that support long-term learning.

Since teachers and students are often not of the same cultural backgrounds, the need for teachers to cross culture bounds was expressed repeatedly in recent research (Ensign, 2003; Gay, 2002; Talbert-Johnson, 2006). Research like this study that explores culturally responsive pedagogies, can equip educators with new ideas to engage all students.

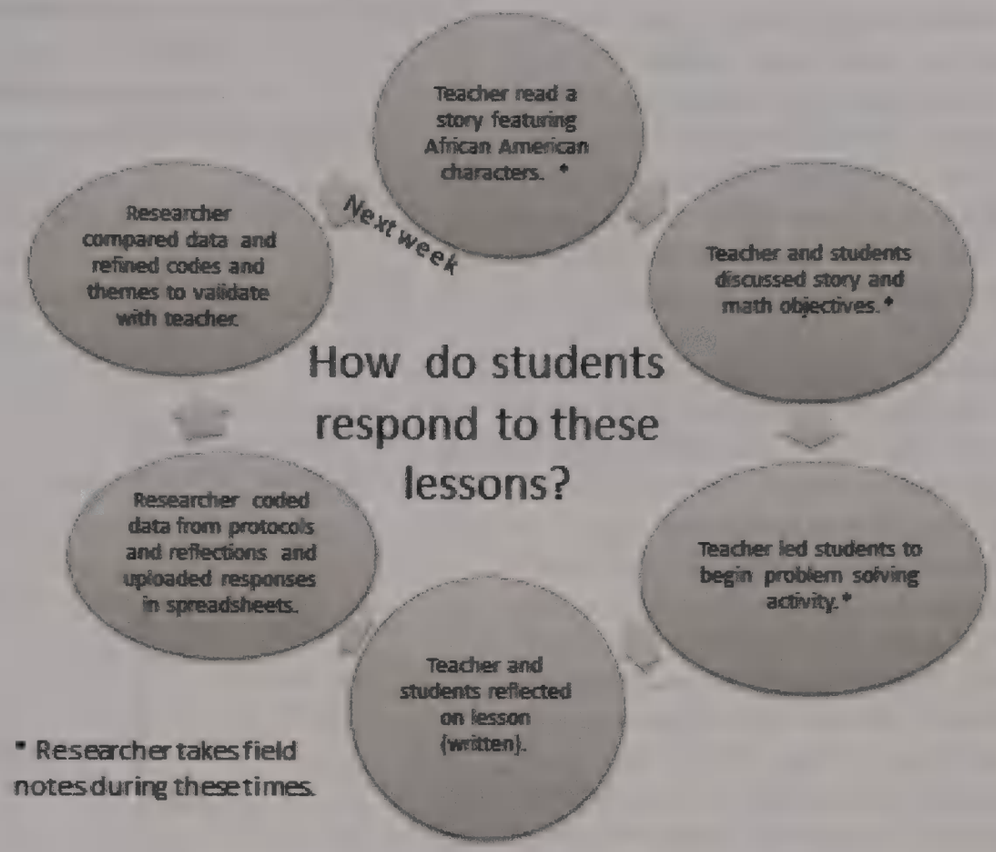

Teacher read a story featuring African American characters. *
Teacher and students discussed story and math objectives. *
Teacher led students to begin problem solving activity. *
Teacher and students reflected on lesson (written).
Researcher coded data from protocols and reflections and uploaded responses in spreadsheets.
Researcher compared data and refined codes and themes to validate with teacher.
Next week
How do students respond to these lessons?
* Researcher takes field notes during these times.

Figure 1. Flowchart for qualitative study based on an action research framework.

Methodology

A qualitative design was appropriate for this study because the focus was on the way students responded to the lessons with a story featuring Black characters. Engagement is a subjective issue that cannot be clearly defined numerically. However, behavioral engagement has certain characteristics that can be observed physically. In a comparative analysis of various instruments to determine student engagement, Fredricks and McColskey (2012) noted that observation was a strong method for gathering data to determine behavior and emotional engagement. They also found that student self-reporting was helpful in gathering evidence of cognitive and emotional engagement. Both of these methods are part of ethnographic research techniques (Fetterman, 2010) and were employed to interpret and describe what I heard and saw in the classroom. Descriptions and interpretations of these behaviors and feelings from differing points of view (the teacher, the researcher, and most importantly the students' reflections) helped validate engagement or disaffection. A qualitative design allowed for multiple avenues of collecting data (numerical, verbal, and visual), providing a more complete picture of how these students responded in the classroom.

A framework of action research in early childhood education was utilized (Castle, 2012). The cooperating teacher and I began with a problem. I examined recent explanations and solutions in the literature and prescribed a plausible solution. We agreed and successfully field-tested the use of stories featuring Black characters in spring with diverse learners. Evidence from these lessons prompted us to create lessons with such stories as the plausible solution we would use in our study (hereafter called the intervention or treatment). Then Mrs. A. implemented the intervention (Castle, 2012) in the classroom. Figure 1 shows my participation as each part of the lesson was conducted.

The Participants

Although all students participated in the study the primary unit of analysis for this article is the responses and engagement (or disaffection) of Black students in two third-grade mathematics classes. These students were parents-identified to the district as Black ($N = 17$). The two classes represented the average demographics and size for the school: nearly half Black, with roughly 22 students in each class. As some of the data relied on the students' ability to communicate in written and verbal form, third-graders, instead of younger students, were chosen for this study.

The cooperating teacher, Mrs. A. (pseudonym) was recommended for this study by her principal because she came from an all-Black urban school with very positive recommendations as a teacher and because of her current

rapport with Black students. She is White, in her twenties, and had taught three years in an urban school. We both view the deficit in state and national scores as an equality of teaching issue (Gutiérrez, 2008; McMillian, 2003). We both experienced Black students disengaging from mathematics (much like McMillian, 2003) in our urban teaching experiences and wanted to determine a teaching strategy that might better engage minority populations.

The Researcher

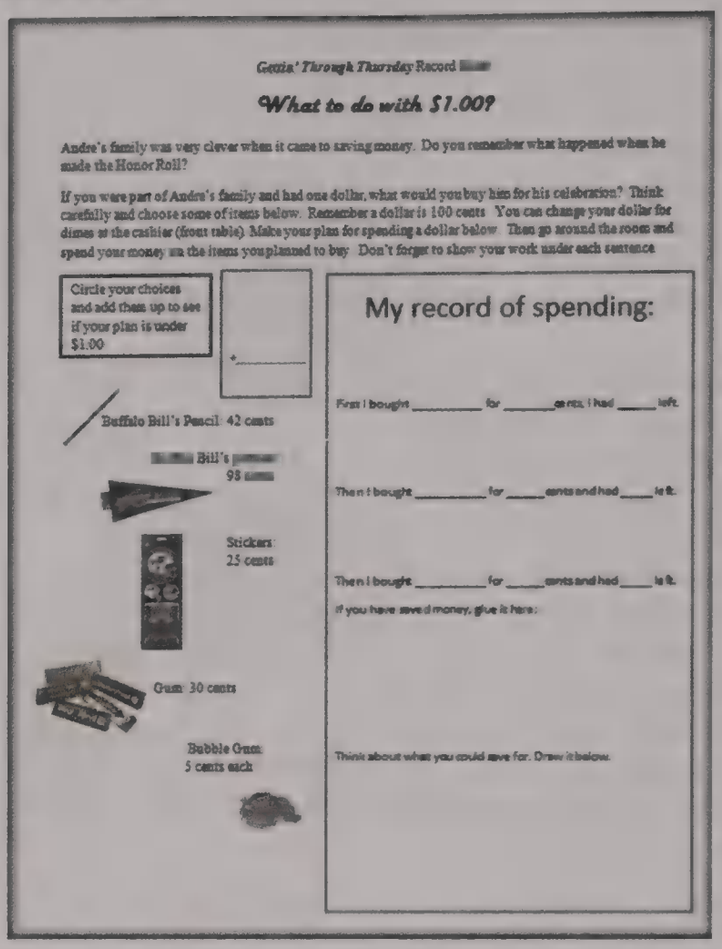
Due to my experience with culturally responsive teaching (12 years in diverse elementary classrooms, 6 of which were in urban classrooms with nearly all minority students), I admit I anticipated positive results from using these stories. To avoid any bias, I was particularly careful to record only what was observed (not implied) to accurately represent what happened in the classroom (Fetterman, 2010). As key informants, the students and teacher helped build construct validity by confirming or correcting my field notes, interpretations, and descriptions intermittently throughout the observations (Yin, 2013). To ensure reliability I used multiple sources of data from the students and teacher (work samples, reflections, interviews, focus groups, etc.). Unfortunately, videotaping was not an option.

Timeline

This study was conducted over the fall semester during the once a week mathematics lessons (typically 90 minutes) using a story featuring Black characters. Twelve lessons using these stories were taught by the teacher (data was not collected on lesson 12 due to time constraints from the scheduled winter party). See the Appendix for a list of the titles and mathematic connections used in this study. The first week of the study was used for the counselor on campus to obtain assent from the students and permission from the parents for participation in the study.

The Treatment: Culturally Responsive Lessons

Each lesson contained three elements; reading the story, a short mathematical discussion connected to the story, and a time of mathematical problem solving or mathematical activity linked to the story. I designed each lesson by aligning the story to a mathematical concept from the state standards with approval and feedback from Mrs. A. The stories chosen for the lessons met the conditions for being quality African American literature as described by Hefflin and Barksdale-Ladd (2001). Most chosen stories were realistic or historical fiction about everyday situations that required some mathematical reasoning or problem solving. The stories lent themselves to mathematical extensions that the students could continue within the class, or practice outside of school. I designed a recording sheet that included the problems connected to the story and space for students


Gettin' Through Thursday Record

What to do with $1.00?

Andre's family was very clever when it came to saving money. Do you remember what happened when he made the Honor Roll?

If you were part of Andre's family and had one dollar, what would you buy him for his celebration? Think carefully and choose some of items below. Remember a dollar is 100 cents. You can change your dollar for dimes at the cashier (front table). Make your plan for spending a dollar below. Then go around the room and spend your money on the items you planned to buy. Don't forget to show your work under each sentence.

Circle your choices and add them up to see if your plan is under $1.00

+ _____

My record of spending:

First I bought _____ for _____ cents. I had _____ left.

Then I bought _____ for _____ cents and had _____ left.

Then I bought _____ for _____ cents and had _____ left.

If you have saved money, glue it here:

Think about what you could save for. Draw it below.

Buffalo Bill's Pencil: 42 cents

[redacted] Bill's [redacted]
98 [redacted]

Stickers
25 cents

Gum: 30 cents

Bubble Gum
5 cents each

Figure 2. Example of a recording sheet (this one is from week four).

to show their work in determining solutions. Figure 2 is one example of the record sheet students used to record their answers.

Data Collection

I observed all 11 lessons twice (once with each class). During these field observations, I attended particularly to how Black students reacted during the story, discussions of mathematical connections to the story, and mathematical activities or problem-solving activities extending from the story. During these observations, I collected data from multiple sources, including field notes, students' work, artifacts, and tallies of noted active responses during story time and discussion. Participation and paying attention were noted as two main aspects of behavioral engagement on of the student engagement instruments analyzed by Fredricks and McColskey (2012). Action initiation, effort, exertion, attempts, persistence intensity, attention, concentration, absorption, and involvement were listed in the conceptualization of engagement in the classroom, by Skinner et al. (2008). Observations included noting consistent eye contact with book or reader, answering questions, volunteering information or raising hands,

Reflection of stories in math: What do I think?

Circle your choice and write if you want to explain.

How do **you** feel about the story this week?	It was boring.	It was ok.	It was interesting	It was awesome.
Did the story help **you** think about math?	Not at all.	It did some.	It did a lot.	
Did the story make **you** think about math in **your** life (like outside of school)?	Not at all.	It did some.	It did a lot.	

Do **you** think **your** teacher should use this story with next year's class?
Yes/No

Why or why not? _____

_____

_____

_____

Does it matter to **you** that the main characters were African American?
Yes/No

Why or why not?_____

_____

_____

Figure 3. The reflection sheet completed by student and teacher for each lesson.

shouting out, acting as the character in the story, as well as effort and persistence in their problem solving.

Students reflected at the end of each lesson in written form to a five-question reflection. The first three questions invited students to respond to their feelings about the story (emotional engagement), and thoughts or attention to math (cognitive engagement) because of the story. The next question asked if they thought the story should be used again for next year's students and to explain why or why not. The last question was not part of this study but was an attempt to gage their personal cultural connection with the story (see the reflection template in Figure 3). Mrs. A. also completed the same reflection to provide her perspective on how she thought students were connecting to the mathematics in that week's lesson. Following the final lesson students also completed an end of the semester questionnaire (used in this study only as qualitative support for their responses to the weekly reflections).

A short vignette. Just after the Thanksgiving break students listened to the story *Lucky Beans* (Birtha, 2010) about a family living during the Great Depression. Like many at that time the main character's (Marshall) father is out of work and they must eat beans for every dinner to economize. Marshall notices a contest for a sewing machine at the local furniture store based on estimating the number of beans in a large jar. He and his mother use proportional reasoning to win the contest.

Students react to the story with laughter as the character talks about always having beans and at his facial expressions. One student reacts as Marshall looks in the window at the sewing machine, "That mamma would go crazy for the sewing machine!" I note that most of the students are responding to the mathematical reasoning that Marshall does with his mother, "Oh, no, he's going to count them all!", "No round it." "They're estimating!" "Put them on plates to count them. Put 100 beans in each because it's easy to count by 100." "Estimate by the size (of

the jar)." "He can use the cups and pour some beans in there." In the second class students had similar statements "Use the empty jars." "You've got to group them and count." "4, 341, but that's all!" and "What if he puts them in groups, then estimates by size?" When his mother's estimate is closest and she wins the contest, students smile, clap, and several said, "I knew it!"

Mrs. A. asks student how they feel this story connected to math. In each classroom students notice estimation, rounding, using multiplication, and using cups and quarts to determine a larger amount. Students are dismissed to their groups and given 16 ounces of red beans, a measuring cup, a pint size baggie, and their recording sheet. Mrs. A. asks them to think about Marshall's plan for estimating from the story and to discuss with their group what they might do to find out how many beans are in her classroom jar.

Most groups decide to fill a cup with beans and then divide them by handfuls to count. Most students group them to count (either in groups of 5, 10, or 20), but one group makes rows and columns. Another student tells me proudly that they used repeated addition and have 134 beans in a cup. When students realize this amount only filled about half of the baggy, most are frustrated and want to fill the baggy and count all of them again. Mrs. A. asks if there is an easier way. Marcus (pseudonym) raises his hand frantically. "Why not just double the amount?" Students seem to reason through this idea and determine the amount of beans in a pint. Now they have to estimate how many beans are in the class jar. After one student takes his baggie and holds it up next to the jar to visually estimate how many baggies would fit in the jar, all groups do the same. Most use repeated addition to determine their estimate. We hoped students would ask how big the jar was but no one does. Mrs. A. reveals that the jar had about 1,300 beans. Some students are cheering (those closer to the amount), and a few are pouting in each class. Mrs. A. tells them they can try again on the back part of their recording sheet, which asks them to estimate the amounts of beans needed for dinner.

As I observed students working on their own, I note they are on task until she rings the bell (they only had about 5 minutes after the group work to finish). Students then complete their reflection before leaving.

Data Analysis

Data from the observations during the story time, discussion time, and problem-solving time were coded weekly for common behaviors among students (examples: making eye contact, mimicking the story character, stating direct connections to math done in the story, participating physically in problem solving). The codes that emerged the most were used to build themes to explain (Castle, 2012) how Black students responded to doing mathematics as relating to the story read earlier in class. By constant comparison analysis (Glaser, 1965) of the data week to week, I formed working interpretations and descriptions throughout the study. The codes and themes were discussed with Mrs. A. bi-weekly for validation. The results of the data analysis were also validated by a final interview with Mrs. A.

Observations and field notes were transcribed and input into computerized spreadsheets on the day of observations. Information from the weekly student and teacher reflection sheets was uploaded into these spreadsheets by assigning a numerical code for all possible circled responses from the multiple-choice section (and transcribing handwritten text from the open-ended questions). The numerical code assigned answers from the most positive choice to least positive (4–0).

The results from the quantitative data supported and strengthened the qualitative results. The numerical charts further demonstrated how the qualitative findings were validated over the course of the study using frequency, percentages, and counts.

For further analysis, the results of the study were compared to the research. First the results we compared to the results found for disengagement in the research with African American students (McMillian, 2003; Steele, 1992) and contrasted sharply with students being actively (behaviorally) engaged and reflecting that they were cognitively engaged, as evidenced by their explanations of solutions and responses during mathematical discussion of connections between the story and math standard of the week. When compared to the researched characteristics of engagement, we agreed the data supports that all students, particularly Black students, were behaviorally, emotionally, and cognitively engaged. When compared with research from Sciarra and Seirup (2008) about engagement positively impacting mathematics achievement for Black students, Mrs. A. pointed to improved grades and test scores, although we agreed it could not conclusively be due to these lessons alone.

The results of the study were also compared with positive findings in the research on using literature to teach mathematics. The main benefits described in the research were: engagement in the lesson, extended effort and time in mathematical thinking, active problem solving, and positive dispositions toward doing mathematics (Furner, Yahya, & Duffy, 2005; Hong, 1996; Jennings et al., 1992; Keat & Wilburne, 2009; Shatzer, 2008; Tucker et al., 2010). All of these were affirmed in the study and are discussed in the results.

Results

The behavioral engagement was determined by field notes during the story and discussion times. The emotional engagement was determined by their responses to the first reflection question that asked how they felt about the story and validated by both Mrs. A. (her responses to this reflection question) and agreement upon developing themes from the field notes. This mirrored the findings of using stories with Black characters to teach Black students found in recent research: that these students responded with behavioral and emotional engagement (Altieri, 1993; Hefflin, 2002; Hefflin & Barksdale-Ladd, 2001; Tatum, 2006). Cognitive engagement as evidenced by student responses to reflection questions two and three was confirmed by their responses during discussion and problem solving, noted in observation protocols. The details from each source (students, Mrs. A and myself) are described below.

Results from Researcher's Observations

As stated earlier, students were observed throughout each part of the lesson. To understand how Black students responded to the story I recorded and analyzed their behaviors during the reading of the story, as well as their answers (on reflection question one) about how they felt about the story. To determine how students responded to the mathematical concepts in the lesson I reviewed the results of observations during the discussion time (when they offered ideas and explanations of how the story connected to the math standard of the week), and problem-solving time.

Story time. As I sat off to the back corner of the carpet each week I recorded students' actions and tried to record comments, as well as notice posture and eye contact throughout the story. Students often mimicked the behavior of the character (some examples of these were counting coins, stirring the soup, acting with similar facial expression, and swinging a bat). Several students, especially boys, were observed leaning in or sitting on heels to better see the story. Many of the students responded verbally (words, exclamations of reaction, or advice to the character). Some examples (from the first class) of the verbal reactions from the first story about the runner Wilma Rudolph were: "Ohhh no," "Whoa!", "Wow! No way!", "She's gooood," "Don't give up," and "You better win girl!" Data analysis shows the average number of Black students engaged weekly in verbal responses was 76.6%. A careful count of eye contact at the beginning, middle, and end of the story time showed a weekly average of 92.5% of them also had consistent eye contact throughout the story. The data supports that students were behaviorally and emotionally engaged.

Students responses to reflection question one (and the teacher's response to this question) validate students' pleasure with the stories. I conclude that students who would rate the story as boring would equate with disaffection and those with positive ratings of the story and recommendations to use the story for next year's students equated with being engaged emotionally to some degree. This is covered in the student section.

Discussion time. While students responded to the teacher's invitation to make connections to their mathematics objective I recorded the specific statements of the Black students. The number of statements was tallied for each student per week and analyzed as a whole to determine common themes. Not all comments were directly related to mathematics so the percentages of comments that did relate to mathematics were calculated for each lesson. Also, the number of students making comments related to mathematics was calculated by percentage for each lesson. Table 1 displays the results of these calculations.

Students' statements that were directly related to math were coded and counted for frequency by class. Analysis for common math-related statements led to the development of several codes: quantity (how many) or counting, measurement, reasoning or problem solving, and mathematical operations (addition, subtraction, multiplication, and division). Frequency counts were totaled for each code by lesson and were compared to the weekly objective to determine trends in the sophistication of their connections or number of students responding over time. Comments coded as "connections to mathematical operations" had the highest frequency count with 39 statements, most of which correlated directly to the objective for the week. For example, the objective for week

Table 1
Data Analysis for Speaking Out and Eye Contact for Black Students During Story Time from Both Classes Combined

Student Behaviors	wk1	wk2	wk3	wk4	wk5	wk6	wk7	wk8	wk9	wk10	wk11
Percent of students commenting	81.3	88.2	82.3	64.7	75	58.8	75	76.5	64.7	76.5	100
Total comments	36	75	56	30	64	26	41	67	33	77	71
Maintaining eye contact	n/a	100	87.5	100	86.6	82.3	86.6	94.1	100	94.1	93.8

two (lesson two) was: students will model addition and subtraction using pictures, words, or numbers. "Add his score," "Use the number he scored and the rest of his team, and "Add up to see how much he grew" represented a sample of these statements that related to addition from the story and matched the objective of week two. Much of the discussion (an average of 75% of comments made during this weekly time) centered on mathematically connecting the objective with the story. These results point to cognitive engagement because they were thinking about the math from the story to make connections.

Problem-solving time. As the researcher, I attempted to record students' comments and behaviors as much as possible using a problem-solving protocol while all students worked at tables simultaneously. The protocols were reviewed weekly and coded for common elements. Codes were refined by comparing each week's problem-solving protocol.

The first theme of students' persistence was defined by actions and statements that established a student's effort in persisting to solve a problem by getting help, trying more than one strategy to solve a problem, or reworking the problem for accuracy. The results confirm that these students were cognitively engaged by their willingness to persist.

The second theme of students' understanding of mathematical concepts while problem solving expanded from week to week. Understanding, in contrast to memorization, demonstrated how students were more involved with the problem than merely following problem-solving steps and indicated students' ability to defend their work. Again, this is further evidence that students were responding to the problems related to the story with cognitive engagement.

Results from the Students' Data

All students had the opportunity to share their thoughts in the weekly reflections. The first question of the reflection asked students to circle how they felt about the story (see Figure 3). Data analysis of student responses demonstrated that the average response for every story was between interesting and awesome (the two most positive choices). Only one student rated a story as boring (they did so twice). Students also were asked if they would recommend that the teacher use this story again with next year's students, and if so, why. All students responded positively to this question, with Black and Hispanic students agreeing with a 96% approval for all stories to be used again. White students also felt the stories should be used again with an 89% approval for all stories. When the written explanations were analyzed, Black students described two main reasons for their affirmation: enjoyment and learning mathematics. We can assume if they recommend the use of these stories for these reasons that they thought the stories were enjoyable and lead to their own learning (further evidence of emotional and cognitive engagement).

The second and third questions on this reflection sheet gave students another occasion (besides the discussion time) to express the degree to which they felt the story made them think about mathematics and invited them to explain. Giving students more options for expression allowed me to better understand how the story influenced students to think about mathematics. The data responses for reflection question two and three are addressed separately in this section. These questions were designed to capture students' perspectives about how the stories influenced their thinking about mathematics.

The second reflection question asked students how they felt the story made them think about mathematics. Responses to this question were numerically coded and were entered into a spreadsheet weekly according to class section. The value of one was assigned to the most negative response (1 = not at all), a two for a more neutral response (2 = it did some), and a three to the positive response (3 = it did a lot). The data were analyzed to determine the degree to which Black students felt the story helped them think about mathematics. Both class sections' average responses were very close: an average rating of 2.56 for section one, and 2.66 for section two demonstrated reliability within the data and the question. The range of overall weekly responses, 2.38–2.91, indicated that these students felt the stories helped them think about mathematics either some or a lot each week. Just over 94% of responses indicated that the story had some degree of helping them think about mathematics. Figure 4 shows the resulting weekly averages of the responses from all students. The data from all students are displayed to show how Black students answered most affirmatively and how all students were positively responding. These data strongly support that students felt they were cognitively engagement in mathematics to some degree in these lessons each week.

The third reflection question was designed to capture if and how students felt the stories influenced their thinking about mathematics outside of school. The responses were numerically coded in the same manner as the second reflection question. Comparative analysis between the two class sections revealed similar results. The overall average rating of how Black students felt the stories helped them think about mathematics outside of school was 2.32; just above the numerical code for "It did some."

One hundred seventy-three responses were given for this question. Of those, 134 were coded a 2, or 3, demonstrating that 77.4% of students felt the story helped them think

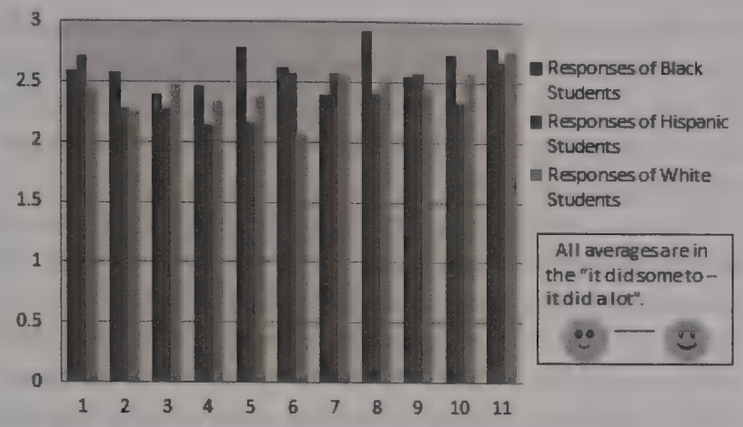


Figure 4. Data results from reflection question two: comparison by race. Note: Averages determined from weekly student data, aggregated by race as parent identified to the school district.

about mathematics outside of school to some degree. These results support how they were cognitively engaged in mathematical thinking.

Data from their work validated the researcher's theme of persistence. Out of the 180 record sheets, 57% of students rewrote their attempt to at least one problem (evidenced by erasure or marked-out work) and 71% of students tried more than one way to solve the problem each week. The total who tried either (to rewrite their work or tried another way to solve the problem) was 91%. Along with confirmation from their teacher that they were working much longer than with regular lessons from the workbook, this is further evidence that students were behaviorally engaged in doing mathematics.

Results from the Teacher's Reflections

Mrs. A. responded to the same reflection questions as the students for each week's story. Her responses were analyzed just like the students. She felt students reacted positively to all stories and also circled the choices as interesting or awesome. Her interview data described how students talked about the story later in the day or week, especially at recess or during the math lessons that followed with the same concepts.

The analysis defined both the teacher and the students as having affirmative responses to how students felt the story helped them to think about mathematics. The small comparison discrepancies were in relation to the degree of how much they thought the story helped them think about mathematics: either some or a lot. So evidence that students were behaviorally, emotionally, and cognitively engaged is triangulated by the students', teacher's, and researcher's data.

Implications and Discussion

Since this study relied on research conducted with students and a teacher in an elementary classroom, results of this study lead to many implications that affect elementary students and teachers. There are also implications that connect to teacher preparation for diverse classrooms. I explain specific implications through reflection of the findings, and application for students, teachers, and instructors in teacher preparation.

Implications for Students

One of the main reasons for using a story in teaching mathematics is to facilitate thinking about mathematical concepts (Whitin & Whitin, 2004; Wilburne, Keat, & Napoli, 2011). Nearly all students in this study responded to the second reflection question that the stories helped them think about mathematics to some degree. "The world of a child involves patterns, problem-solving, communication, and connections" (Moyer, 2000, p. 255). Black students in this study confirmed the connection between the story and mathematical problem solving in their discussion time with Mrs. A. just after the story. Since students talked about mathematical connections to the story and reflected that the story helped them think about mathematics, it seems logical that using stories featuring Black characters is one way to get these students behaviorally, emotionally, and cognitively engaged in mathematical problem solving.

Students must discover that mathematics is related to their everyday lives (Civil, 2007; Putnam, 1992). The lessons in this study helped Black students realize how mathematics relates to everyday life by seeing how the characters (reflecting their race) used mathematics or solved problems with mathematical thinking in these stories. More than 77% of them (student average for all weeks) responded that these stories helped them think about mathematics outside of the classroom. This reiterates how students connected some mathematical actions or circumstances from the story to their own life. In discussion of math operations to a story about cooking a special dinner, Maya said, "So like when you cook, you have instructions, and you have to know how much to use. You can add when you have something small like only a $\frac{1}{2}$ cup (measuring cup) and it says you need 4 cups." When students of color do not see themselves reflected in the stories they read year after year, they become disengaged or unattached to the characters (Hefflin & Barksdale-Ladd, 2001). Week after week, these stories demonstrated a way for Black students to see mathematics as a personal and real-life activity.

Another implication for students is how the stories gave all students the same background knowledge for solving the related mathematical problems. Teachers often cite background knowledge as a weakness for students of color

when they attempt word problems. One of the benefits of using a story to teach mathematics is the way a story creates a shared experience for the students and teacher (Furner et al., 2005). When asked if the story used that day should be used in the same manner next year, Black students responded 96% of the time affirmatively. Their main two reasons for using the stories were enjoyment and learning mathematics. Marcus said, "Yes because you have to add sometimes and subtract, I liked to add (from a story activity)." In response to a story about managing finances, Malik said, "Yes, because it can help you in math. You can see how you get through days till the check comes. Most people wait to get their check." Using stories promoted interaction and discussion so students understood what was going on in the related mathematical problems and activities.

When a story is used as the basis for mathematical discussion, the possible outcomes could also include a sense of belonging and a sense of community (Walshaw & Anthony, 2008). Many stories created space for student connections to each other. In particular, the story about doing without until Mom was paid, Getting' Through Thursday (Cooper, 1998), resonated with nearly all students. Many students connected to sports, family dynamics, or hardship in the stories. Black students also seemed confident and excited to do the word problems related to the story with their diverse small group. They often reworked problems during practice time and tried different methods to solve problems together that indicated some community and sense of belonging due to these story-related problems.

Implications for Teachers

"The question is not whether all students can succeed in mathematics but whether the adults organizing mathematics learning opportunities can alter traditional beliefs and practices to promote success for all" (National Council of Teachers of Mathematics, 2014, p. 61). Implications from this study may assist teachers in improving not only how to engage more students; they may also improve their instruction for higher-level mathematics skills, like persistence in reasoning and problem solving for more students. To help students improve their problem-solving skills, teachers should expose students to a variety of problems, often over an extended period of time (Schoenfeld, 1992). Assigning students weekly problems connected to these stories gave students consistent opportunities to practice their problem-solving skills in an engaging manner each week. Students connected to the story so well that Mrs. A. and I agreed that some students excelled in problem solving due to the effort given to thinking and trying solutions during these lessons. We both attribute this to their engagement and/or connection with the story. Sometimes it seemed that students just felt like they were letting the character down if they could not figure out the related problem.

Teachers should design meaningful problems to stimulate students' interest and encourage the disposition of persistence in thinking mathematically. Teachers should be encouraged by these results to design varied tasks and problems in the context of stories that reflect students' culture and race. Although student achievement was not measured, students responded to word problems with mathematical language, reasoning, and understanding as they persisted in solving problems connected to the story. These mathematical behaviors go beyond the memorization of math facts and procedures. Mrs. A. and I also conclude that in this study, the advanced practices of mathematical speech, reasoning, and understanding are integral to student success. Utilization of these stories proved more powerful in engaging students in doing, thinking, and reflecting about mathematics than Mrs. A. had ever seen with lessons from the textbook.

Teachers should also value how the perspective of the students is crucial in understanding how to effectively teach. Mrs. A. understood from the reflection data that students were really thinking about mathematics in connection with the stories. Howard (2001) found that classroom teachers "learned critical insights into the dynamics of young African American learners" (p. 145) after reading his account of their responses to questions about teaching. Giving students a voice informs the teacher. This voice must be heard in order for the teacher to construct effective lessons that account for students' preferred activities and learning styles. So along with using stories that respond to students' home life, another implication of this study is how the teacher can elicit the students' points of view and then utilize these insights to design engaging and meaningful instruction.

Implications for Instructors in Teacher Education

This study could be viewed as a model for teaching preservice teachers how to use multicultural stories to teach mathematics while being culturally responsive (connection to and affirming nondominant culture) to a classroom of diverse students. Ladson-Billings (2000) indicated, "As a group, African Americans have been told systematically and consistently that they are inferior, that they are incapable of high academic achievement" (p. 208). Using stories with successful characters of students' own race may help the student understand that the teacher sees their race and culture as successful. Most teachers admit that their

preparation program did little or nothing to prepare them to work with diverse classrooms (Talbert-Johnson, 2006). Using culturally responsive stories is one way to prepare teachers to think about their students' point of view, reflect on their own bias or unconscious racist practices, and to prepare them for teaching diverse populations.

As instructors in teacher education know, teachers must have empathy and understanding of the cultures, race, and ethnicities represented in their classroom. These dispositions are essential to being culturally responsive to students but are not in the content of most teacher-preparation programs (Gay, 2002). This strategy provides preservice teachers with experiences to reflect on that may build understanding and empathy, as well as demonstrate to their students an affirmation for cultures different from their own.

Future Research

Further research should be conducted in similar diverse classrooms that explores the perspectives of all students (not just that of Black students) to understand if and how other students benefit from mathematics lessons that incorporate stories featuring Black characters. Perhaps there are implications of empathy, understanding, and social justice than can be gained from research with all students' points of view. What do other students gain from learning mathematics with these stories? Do the dynamics of the classroom change? If so how? How does the dominant culture react to stories that touch on social inequalities or expose racism? Do the stories we choose really reflect students' culture? Research in this venue may lead to designing instructions that are more integrated and aims to teach comprehensive skills like critical thinking and self-reflection.

Due to the success of this study, I am reviewing all student data for another article, as well as designing a study with stories featuring Hispanic characters. I plan to model this practice with my student teachers and I hope this study challenges you to purposefully use stories to connect students to thinking and reasoning, and to each other.

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Authors' Note

Keywords: elementary mathematics, equitable practice, culturally responsive pedagogy, qualitative research, student perspective.

Appendix

Story Title	Mathematical Connections
Wilma Unlimited (Krull, 1996)	Measuring distance, rounding, devising an exercise routine (addition, multiplication, subtraction)
Salt in His Shoes (Jordan, 2003)	Scoring (addition) real and possible, graphing, open-ended questions
Gettin' Through Thursday (Cooper, 1998)	Using money in story context (adding/subtraction) budgeting a meal, financial literacy
Just Like Josh Gibson (Johnson, 2007)	Measuring distance, graphing Three-digit subtraction from his statistics (real life)
Auntee Enda (Smothers, 2001)	Building arrays, multiplication Portions, ratios
My Rows and Piles of Coins (Mollel, 1999)	Building arrays, grouping in sets Counting, using money, financial literacy
Chicken Chasing Queen of Lamar County (Harrington, 2007)	Using array to determine multiplication fact Multiples of chicks per hen, working with dozen Open-ended problem solving
Beatrice's Goat (McBrier, 2012)	Designing a pen for goats, area and perimeter Word problems (multiplication/division) from the story
Lucky Beans (Birtha, 2010)	Estimation, proportional reasoning Questions from the story using five-digit subtraction, multiplication, equivalent amounts (fractions) Devising a plan for spending
Sweet Potato Pie (Lindsey, 2003)	Devising a plan (to sell pies—determine expense vs. profit, open ended, must save the family farm) Multiplication/division with visuals related to the story Fractional reasoning about size
Mim's Christmas Jam (Pinkney, 2001)	Drawing fractions Seeing fractions (as parts of a set)
Read but no activity due to winter party.	Using a recipe with limited tools (equivalent fractions) Multiplication/division problems from the story Designing train track (using fractional pieces to make amount equivalent to six wholes)

What Is a Fraction? Developing Fraction Understanding in Prospective Elementary Teachers

Stacy Reeder
University of Oklahoma

Juliana Utley
Oklahoma State University

Classroom teachers need a well-developed deep understanding of fractions and pedagogic practices so they can provide meaningful experiences for students to explore and construct ideas about fractions. This study sought to examine prospective elementary teachers' understandings of fraction by focusing specifically on their use of fractions meanings and interpretations. Results indicated that prospective elementary teachers bring with them to their final methods course a limited understanding of fractions and that experiences in methods courses resulted only in minor improvement of those limited understandings. The limited part-whole understanding of fractions that prospective elementary teachers entered the course with was resilient. The implications of this study suggest a need for prospective elementary teachers to continue to develop their conceptual understanding of fractions and for changes to the content and instructional strategies of mathematics content courses designed for prospective elementary teachers.

The difficult and complex task of teaching mathematics for understanding is often challenged and complicated by the fact that many teachers' experiences with mathematics as learners result in their limited understandings of important concepts. Prior to beginning education or mathematics methods coursework, prospective teachers have spent years learning mathematics from teachers whose pedagogical practices primarily reflect a traditional orientation focused on procedural understanding rather than a balanced approach that attends to both conceptual understanding and procedural fluency (National Research Council, 2001).

Research supports the idea that teaching and learning fraction concepts is a difficult and complex undertaking (Ball, 1993; Harvey, 2012; Lamon, 2012; Ma, 1999; Newton, 2008). Charalambous and Pitta-Pantazi (2005) and Harvey (2012) assert that the teaching and learning of fractions has traditionally been problematic while Newstead and Murray (1998) suggest that it is well documented that fractions are among the most complex mathematical concepts that elementary students encounter. Further research reveals that there are several factors that contribute to students' challenges with developing deep conceptual understanding of and procedural fluency with fractions. According to the literature, either the challenge of learning fractions with deep understanding lies in the inherent nature of fractions and the multifaceted construct of fractions or it is due to the instructional approaches for teaching fractions that are employed by teachers (e.g., Behr, Harel, Post, & Lesh, 1992; Brousseau, Brousseau, & Warfield, 2004; Cramer & Whitney, 2010; Kieren, 1988, 1993; Lamon, 2001; McNamara & Shaughnessy, 2010). Many purport that the problem is the confluence of these factors and that the problem is further complicated by the fact that many prospective and in-service teachers have a limited and incomplete understanding of fractions themselves (Ball, 1990; Becker & Lin, 2005; Chinnappan & Forrester, 2014; Cramer, Post, & del Mas, 2002; Harvey, 2012; Ma, 1999; Wu, 2001).

Background

Research has shown that prospective elementary teachers' understanding of fraction content knowledge is not as strong as it should be (Ball, 1990; Cramer et al., 2002; Luo, Lo, & Leu, 2011; Tobias, 2013; Utley & Reeder, 2012). Studies further revealed that prospective elementary teachers have difficulties with the meaning of fractions (Ball, 1990), have difficulty carrying out fractional computations correctly (Becker & Lin, 2005), and have challenges explaining fraction concepts to children (Chinnappan, 2000). This challenge for prospective teachers likely lies in the fact that they themselves have not had sufficient opportunities to learn fractions with depth and develop understandings of the multiple constructs of fractions. Decades of research on fraction teaching and learning highlights why prospective teachers may have a limited understanding of fractions.

In 1988, Kieren reported that middle level students in the United States rely heavily on their rote memory of rules and tricks to solve fraction problems. Since that time, there have been calls for change in fraction instruction to move from procedural instruction to pedagogic practices aimed at developing deep conceptual understanding (Lamon, 2012; Van de Walle, 2007). However, helping intermediate and middle level students develop a rich number sense about and understanding of fractions rather than relying on procedures and tricks is not an easy undertaking.

> Helping children to develop a deep and broad understanding of the rational numbers as well as the ability to compute with them and to reason flexibly with them is not as easy as merely teaching a unit on each of the interpretations. It is far more complex than that! There are a number of central or core ideas that must be addressed in instruction (Lamon, 2005, p. 8).

Lamon's claim that teaching fraction concepts with depth is a complex endeavor necessitates an understanding of the work on fraction meanings and constructs. Kieren (1976) proposed that the concept of fractions consist of several subconstructs or meanings thus revealing the complexity of fraction understanding. He suggested that one must understand each of the subcontracts, as well as understand them jointly, to have a general understanding of fractions. In his initial work, Kieren identified four meanings for fractions: measure, ratio, quotient, and operator. Originally, the notion of part-whole relationship was embedded in the other four meanings, serving as a basis for the development of the other subconstructs, and as such was not included in the list as a separate construct. Later, Behr, Lesh, Post, and Silver (1983) expanded the work of Kieren recommending that the part-whole relationship, seen by Kieren as a basis for the other four meanings, be considered a distinct subconstruct of fractions. Their work established the part-whole meaning of fractions as a distinct subconstruct of fractions and further connected it with the notion of portioning. Charalambous and Pitta-Pantazi (2005) suggested that the examination of the work of Behr et al. revealed that "the part-whole subconstruct of rational numbers, along with the process of partitioning, should be considered fundamental for developing understanding of the four subordinated constructs of fractions" (p. 234). Others (Lamon, 1996, 2012; Mack, 2001; Simon, 1993; Tobias, 2013) have since suggested that conceptualizing the whole is important for understanding a variety of mathematical concepts including contextualizing situations, understanding procedures, and interpreting solutions.

The assumption that conceptualizing the whole and understanding part-whole relationships is fundamental to many important mathematical concepts including the four constructs of fractions identified by Kieren (1976) and the operations with fractions connected to the part-whole relationship hypothesized by Behr et al. (1983) explains why the notion of part-whole occupies a significant place in curricular materials throughout the world. Lamon (2012) suggested that one question that naturally arises regarding teaching the five interpretations of fractions (part-whole, measure, operator, quotient, and ratio) is "should we teach one interpretation in depth or should instruction include all of the interpretations" (p. 257) (see Table 1). In response to this question, she explained that this is perhaps not the right question to ask since not all the meanings for fractions "provide equal access to deep understanding and no single interpretation is a panacea" (p. 257). However, Cramer and Whitney (2010) suggested that the part-whole subconstruct is a good place for children to begin to develop an understanding of fractions. While the part-whole meaning of fractions is the most commonly used interpretation in mathematics textbooks, many researchers believe that placing more emphasis on other interpretations would help students gain a better understanding of fractions (e.g., Clarke, Roche, & Mitchell, 2008; Siebert & Gaskin, 2006).

Dienes (1960) suggested that the quality of learning fractions conceptually is likely to improve when students are exposed to a concept represented through different embodiments and in a variety of different situations. If children encounter and work with the same conceptual structure presented in different forms, they can be helped to "gather the mathematical essence of an abstraction" (Dienes, 1967, p. 32). This suggests that although the part-

Table 1
Fraction Sub-Constructs and Interpretations

Interpretation	Example	Meaning
Part-whole	2 parts out of 3 equal parts	Comparison of the number of equal parts (2) out of the total number of equal parts (3) the whole was divided
Measure	$2-\frac{1}{3}$ units	A distance equivalent to 2 intervals of length $\frac{1}{3}$
Operator	$\frac{2}{3}$ of something	Notion of shrinking and enlarging; for example, of something indicates you are carrying out a process of multiplying by 2 and then dividing by 3
Quotient	2 divided by 3	Viewed as a result of division; notion of partitioning or sharing (e.g., amount each person gets when 2 wholes are shared equally among 3 people)
Ratios	2 to 3	Can be a part-part or part-whole relationship based on the context; (part-part: 2 boys to 3 girls; part-whole: 2 boys to 3 total of 3 students)

Note: Framework adapted from Lamon's (2012).

whole relationship for fractions may be the basis for understanding the other fraction constructs, it is not sufficient by itself for deep fraction understanding. It is essential to engage students in with a wider range of models (Clarke et al., 2008) and let them have access to "some alternative form of representation" of fractions besides the area models (Moss & Case, 1999, p. 144) if we want them to develop a deep understanding of fractions. This necessitates that teachers not only need a well-developed and meaningful understanding of fractions themselves but also pedagogic practices that provide experiences for students to explore and construct ideas about fractions.

The purpose of this research was to examine prospective elementary teachers' basic understanding of fractions. While we aim for prospective elementary teachers to have a well-developed, complex understanding of rational number concepts before entering the teaching profession, for the purposes of this study, we were interested in their basic understanding of what a fraction is and if they understand what they have when they have a fraction (e.g., what is 5/6 and how does it compare to other fractions?). Our belief as mathematics educators who are familiar with the research on prospective elementary teachers' content knowledge, and having conducted research on prospective elementary teachers understanding of fractions and operations with fractions, was that the prospective elementary teachers we work with did not have a strong understanding of what we considered basic ideas about fractions. Moreover, that without out a sound understanding of these foundational ideas about fractions, much of what we do related to operations with fractions may not be fruitful. Specifically, this research focused on prospective elementary teachers' understandings of fractions and the impact of the experiences in a mathematics methods course on those understandings.

Method

Participants and Context

The participants in this study were prospective elementary teachers (41 females) enrolled in a fourth- through eighth-grade mathematics methods course. These prospective elementary teachers were in their final year of study and had successfully completed 12 university credit hours of mathematics including a course designed specifically for elementary education majors that addressed foundations of numbers (set theory, numeration, and the real number system), number theory algebraic systems, functions and applications, and probability. As part of this course, prospective elementary teachers were taught rational number concepts, including fractions and operations with fractions, using a traditional algorithmic approach. Additionally, each participant had successfully completed a mathematics methods course focused on grades pre-Kindergarten through third. In this primary mathematics methods course, prospective elementary teachers were involved in minimal fraction experiences that included early fraction concepts via use of the Fraction Kit and the associated Cover-up and Uncover games (Burns, 2001).

Data Collection and Analysis

Data collection involved a focus on both prospective teachers' ability to explain what a fraction is and their ability to identify the largest fraction in a set of three fractions. Specifically, participants were asked to respond to two short answer tasks (see Figure 1). Task 1 focused on prospective elementary teachers understanding of the meaning of fraction and the model they used to support their notion of fraction. Task 2 focused on prospective elementary teachers' ability to identify the largest of three fractions and their explanation of their thinking about their selection.

Data were initially collected the week prior to any instruction on, experiences with, discussion of, or readings on fractions. These tasks were given in class and prospective elementary teachers were provided ample time to complete them. While used as data for this project, these tasks provided a starting point for these prospective elementary teachers to focus on fractions in their weekly reading and as a formative assessment for course instructors. Post-data were also collected in class at the end of the course as students were beginning to prepare for their final exams, thus post-data followed all instruction and experiences related to fractions. Data resulting from both tasks were analyzed by each researcher independently. The data obtained from Task 1 were analyzed and coded using a framework adapted from Lamon's (2012) interpretation and meaning of fractions categories (see Table 1).

Data resulting from Task 2 were scored as either correct or incorrect. Further, each explanation provided for Task 2 was also scored as either correct or incorrect and was subsequently analyzed and coded for themes in the prospective elementary teachers' explanations. Once the independent researcher coding phase was complete, data analysis was compared to determine any differences among the researchers' codes. Any differences were discussed until 100% agreement was reached.

Learning Experiences

The learning experiences provided for the participants in this study as part of their fourth- through eighth-grade mathematics methods course were purposefully designed by the instructors to not mimic the types of experiences that have traditionally focused primarily on procedural

Task 1: Description of a fraction	**Task 2:** Which fraction is the largest?
Instructions: Please respond to the following prompt in writing and include any drawings or examples that support your response. **Prompt:** Suppose a third grader asked you "What is a fraction?" How would you respond?	**Instructions:** Without relying on calculations, please select the largest fraction from among the set of three and explain your reasoning for your selection. Make your selection quickly and then focus on your explanation. **Prompt:** Consider the following three fractions: $\frac{99}{100}$ $\frac{6}{7}$ $\frac{15}{16}$ Which is the largest? Explain your reasoning.

Figure 1. Short answer fraction tasks for data collection.

efficiency. Although this course was not a "content mathematics course" the course was designed with recommendations from the Conference Board of the Mathematical Sciences (CBMS, 2012) in their "The Mathematical Education of Teachers II" document wherein they state "that before beginning to teach, an elementary teacher should study in depth, and from a teacher's perspective, the vast majority of K–5 mathematics, its connections to prekindergarten mathematics, and its connections to Grades 6–8 mathematics" (p. 23). Specifically, the teaching and learning of fractions for this course were designed to provide the prospective elementary teachers opportunities to construct conceptual understanding of fraction ideas and learn them with depth rather than via an algorithmic approach. As such, the activities in this elementary mathematics methods course involved the prospective elementary teachers in a variety of experiences with fractions purposefully selected to develop, with depth, their conceptual understanding of fractions and their pedagogical knowledge about how to foster deep fraction understanding with elementary students.

As part of the ongoing activities of this course, the prospective elementary teachers were engaged in problem solving with fractions, working with hands-on fraction models, creating and modeling of fraction addition, subtraction, multiplication, and division problems, ordering fractions and making connections between models used with fractions and models used with whole number addition, subtraction, multiplication, and division for a period of approximately six weeks out of the 16-week semester. The prospective elementary teachers were asked on a regular basis to present and defend their thinking about problems and tasks as well as consider multiple approaches for solving and explaining paths to solutions. During all work related to fractions, the prospective elementary teachers were required to use multiple representations and interpretations of fractions. For example, students were asked to model and/or draw representations of equivalent fractions and to add to fractions that would require them to draw upon the various fraction interpretations using tools such as fraction tiles, fraction circles, counters (set model), and a number line. Students were required to complete numerous tasks from the *Developing Mathematical Fluency* text (Wheatley & Abshire, 2002) focused on fraction concept understanding and computation. They were also engaged in a variety of tasks involving comparing fractions using fraction benchmarks and were always asked to explain their reasoning in words and with illustrations. Additionally, students were given a fraction computation such as one-half plus one-fourth and asked to write contextual stories that would elicit that computation. As a part of these activities, they had to write stories that elicited the various interpretations of fractions that their students might encounter. The readings and discussion in this class were purposefully focused on the need to utilize interpretations beyond, and in addition to, a part-whole interpretation. The instruction was explicit about the fact that teachers need to help students develop a deep, multifaceted understanding of fractions and that included an understanding of the multiple interpretations and models of fraction.

In addition to explicit instruction and experiences focused on the importance of understanding the multiple interpretations of fractions and the need to utilize a variety of models for the teaching of fractions, developing conceptual understanding of operations with fractions was a

goal. Throughout the course, a concerted effort was made to help prospective elementary teachers make connections to their understandings of whole numbers and operations with whole numbers and develop extensions from their whole number understanding to their newly developing understandings of fractions and operations with fractions. Prospective elementary teachers read a variety of articles focused on teaching fractions developmentally, (e.g., Nowlin, 1996; Reys, Kim, & Bay, 1999; Van de Walle, Karp, & Bay-Williams, 2012; Warrington & Kamii, 1998) establishing fraction benchmarks, and constructivist approaches for multiplication and division of fractions. On a regular basis, the prospective elementary teachers reflected in writing about how the readings influenced them as a learner and how it would influence them as a future teacher. If an article featured a classroom task or activity, it was frequently conducted in class prior to the assigned reading so the prospective elementary teachers could experience it as a learner and then read later about how elementary or middle school students had encountered the task.

Table 2
Frequency of Interpretations, Models, and Contexts Utilized in Task 1

	Pretest	Posttest
Interpretation*		
Part-whole	39	39
Measure	1	0
Operator	0	0
Quotient	5	1
Ratios	0	0
Used multiple interpretations	4	0
None interpretation identified	1	2
Type of Model		
Area	30	23
Length	1	0
Set	0	0
No model	6	17
Symbol	1	1
Context		
Food	14	16
Geometric shapes	16	4
Multiple	0	0
No context	9	23

*A few participants provided multiple interpretations in their responses on the pretest.

Findings

The findings of this study indicated that prospective elementary teachers brought a limited understanding of fractions with them to their mathematics methods courses and unfortunately, what occurred in this course did little to improve those limited understandings. The findings from Task 1 focused on prospective teachers' ability to explain what a fraction is will be presented first. The second section will explore how prospective elementary teachers identified the largest fraction from a group of three and their explanation for their selection.

Meaning and Interpretation of a Fraction

Prospective elementary teachers' responses regarding how they would explain "What is a fraction?" suggest they have a very limited understanding of fractions that is based heavily on a part-whole construct or interpretation of fractions. Prospective elementary teachers' responses were examined and categorized using an adapted framework based on Lamon's (2012) interpretation and meanings of fractions framework (see Table 1). Most the prospective elementary teachers used a part-whole interpretation to explain the idea of fraction before (82.9%) and after (90.2%) the experiences in the mathematics methods course. The prospective elementary teachers' understandings, as expressed in their explanations, varied in their depth as well. Some prospective elementary teachers simply indicated that a fraction is part of a whole while others extended this to include pictures using circles or rectangles limited to an area model. A few prospective elementary teachers expressed slightly more depth related to their part-whole explanations by including varied examples that emphasized the notion of equal sized parts of the whole. Table 2 provides frequency data for responses related to the fraction interpretation and the type of model used in the explanation as well as the context, if any, that was used as part of the explanation whereas Figure 2 provides examples of the responses that included a picture with their explanation. Most prospective teachers' responses pointed to a part-whole interpretation such as "a fraction is a part of a whole. It is a piece of something." The following lengthier explanation was similar to many provided: "A fraction is part of a whole. To better explain, I would then show an example if I had a pie (circle) and I wanted to find 1/2 of it, I would cut the pie into 2 pieces, the bottom number (picture of a circle cut in half) and share the top number (1/2 shaded). I would then use a different example. If I had a pie (circle) and wanted 3/4 cut out or served (pie in 4 parts and another pie in four parts with 3 shaded) again showing the student how the top numbers and bottom numbers correlate." Unfortunately, a few prospective elementary teachers' responses were incomplete or failed to demonstrate any depth in their understanding of how to interpret a fraction. For example, one prospective elementary teacher wrote, "a fraction is a bunch of pieces put together which make up one unit."

Disappointingly, the depth of the prospective elementary teachers' part-whole explanations did not change significantly by the end of the course. The results from the

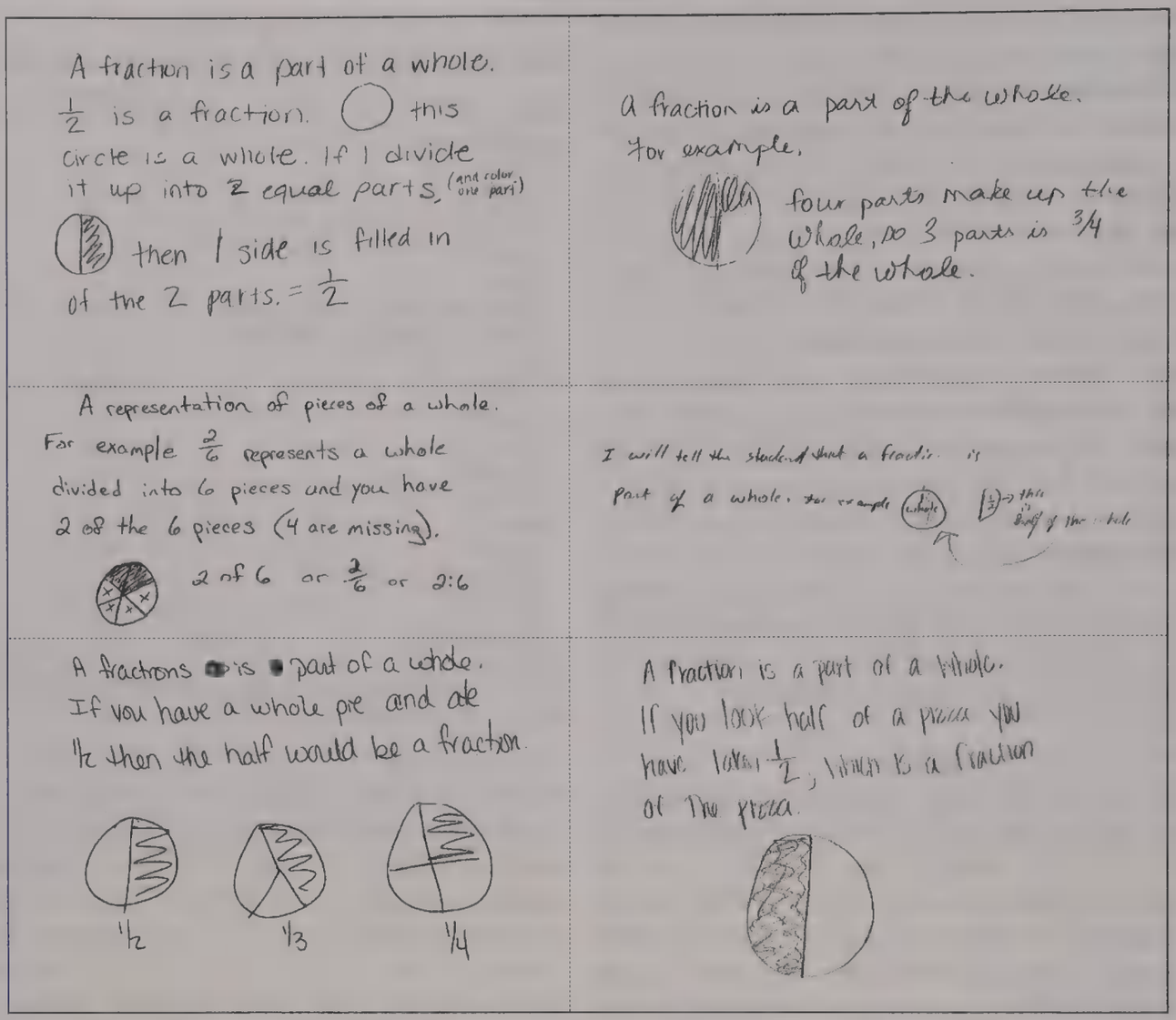
A fraction is a part of a whole.
1/2 is a fraction. this
circle is a whole. If I divide
it up into 2 equal parts, (and color one part)
then 1 side is filled in
of the 2 parts. = 1/2
A fraction is a part of the whole.
For example,
four parts make up the
whole, so 3 parts is 3/4
of the whole.
A representation of pieces of a whole.
For example 2/6 represents a whole
divided into 6 pieces and you have
2 of the 6 pieces (4 are missing).
2 of 6 or 2/6 or 2:6
A fractions is part of a whole.
If you have a whole pie and ate
1/2 then the half would be a fraction.
1/2
1/3
1/4
A fraction is a part of a whole.
If you took half of a pizza you
have taken 1/2, which is a fraction
of the pizza.

Figure 2. Samples of post-course prospective elementary teachers' responses to "what is a fraction" prompt that included pictorial representations.

post data collection revealed that 33 (80.5%) of the prospective elementary teachers had the same part-whole interpretation pre and post. Given that the National Council of Teachers of Mathematics (NCTM, 2000) points out the need for children to understand and represent fractions that are commonly used in contexts such as pizzas, cooking, and measurement as well as seeing them as numbers, it was unfortunate to note that this group of prospective teachers' interpretations and explanations were limited in their use of context as well. Analysis of the prospective elementary teachers' responses revealed that less than half (pre—36.6%; post—39.0%) of the prospective teachers utilized images of food, typically pizza or pie, as a context. It should also be noted that, although a variety of contexts were utilized in class, no prospective elementary teacher used a real-world context other than food.

Relative Size of a Fraction

Prior to the methods course, slightly less than one-half (46% of prospective elementary teachers correctly chose 99/100 as the largest of the three fractions (see Table 3). Three themes emerged from analyzing responses of 99/100 including size of the numbers (pre $n = 3$ and post $n = 9$), comparing the missing amount (pre $n = 14$ and post $n = 18$) related to the fractional quantity, and relating the fractions to a percentage or decimal (pre $n = 3$ and post $n = 0$). In terms of size of numbers, sample responses included "because they have larger numbers" or "because they are the largest numbers. I really have no clue, but classroom age students may have the same reasoning." The participants who selected 99/100 accurately but utilized a size of the numbers argument for that decision seem to be basing their thinking on whole number understanding wherein the larger the number denotes a larger quantity. Most of these accurate responses included a reason that focused on comparing the missing amount of each fraction. For example, one prospective elementary teacher stated, "because in 99/100 the 1/100 of the whole that is missing is smaller than the 1/7 or 1/16 because they are larger parts of

Table 3
Frequency Count of Responses: Which Is Largest? (n = 41)

Response	Pre *n* (%)	Post *n* (%)
99/100	19 (46)	28 (68)
6/7	12 (29)	9 (22)
15/16	2 (5)	0 (0)
All the same	8 (20)	4 (10)

the whole." Three of the correct responses included an explanation that related the fraction to a percentage to compare the fractions. For example, one prospective teacher responded, "if you have a large pizza and you divide it into 100 pieces then having 99 of them would be most of the pizza. Also 99/100 is the same as 99% so you would eat all but 1% of the pizza." Another student shared "because there are more parts in 100 than in 7 or 16, there is a greater chance of there being a larger fraction. In 99/100—99% of the whole is accounted for. In 6/7—about 85% of the whole is accounted for, and in 15/16—about 90% of the whole is accounted for." Additionally, three of the correct choices of 99/100 included an incorrect rationale by the prospective elementary teachers. One prospective elementary teacher suggested that 99/100 was largest but "was not really sure." She indicated that she "picked 99/100 because having 100 pieces would be larger than 7 or 16 pieces of a fraction. But I also think that they could be equal but I am not sure about that either."

Examination of incorrect responses prior to the methods course revealed that a little more than one-fourth ($n = 12$) indicated that 6/7 was the largest of the three fractions. The prospective elementary teachers (pre $n = 7$ and post $n = 8$) primarily focused on the size of a unit fraction based on the size of the denominator referring to a "rule" they were taught in elementary school—"the smaller the number the bigger the pieces." One prospective teacher wrote "In fractions, the larger the bottom number is, the smaller the fraction is." Relying on this vague recollection of a rule without having a deep understanding of what this means and how to apply it to situations, these prospective teachers were left with little ability to determine correctly among the largest of three fractions. Further, a few prospective elementary teachers indicated they "really just guessed" while one prospective teacher focused on how much "space would be taken up" incorrectly suggesting that if you drew them "6/7 takes up more room." Two prospective teachers chose 15/16 but their reasoning was based on guessing. It should be noted that one of the two who guessed also indicated they did not "know because they all look nearly equal." About 20% ($n = 8$) of prospective elementary teachers felt that all three fractions were the "same size." This group of prospective elementary teachers tended to have what Cramer et al. (2002) called "residual thinking" wherein they focused on the fact that each fraction was "one piece away from the whole," ignoring the size of this piece, or they focused on the pattern that each fraction had a numerator that was one unit different less than the denominator.

At the end of the mathematics methods course, slightly more than two-thirds ($n = 28$) correctly chose the largest fraction. The majority of these prospective elementary teachers focused on the "size" of missing piece to provide an accurate explanation. One prospective elementary teacher provided no explanation for why she chose 99/100, one student accurately related her response to percentages, and one student used a visual to provide an accurate explanation. While they made a correct choice of 99/100, four prospective elementary teachers had incomplete explanations that did not reveal whether they had a deep understanding of comparing fractions and another two had incorrect explanations. For instance, one prospective elementary teacher's incorrect explanation was that "100 is a larger whole than 7 or 16."

Nearly one-third of the prospective elementary teachers still had inaccurate responses when asked to identify which of the three fractions was the largest. The prospective elementary teachers who incorrectly answered 6/7 focused solely on the size of the unit fraction ignoring everything else. Sample responses included "because the parts are bigger, the whole is only divided in 7 pieces" or "because it has the largest pieces, 100 pieces would be really small." These responses seem to relate to the "rule" that is often taught in elementary classrooms mentioned previously, that "the smaller the denominator the larger the piece." Analysis of the responses of those who answered incorrectly that all three fractions were the same revealed a focus on the fact that all fractions had one missing piece; the prospective teachers ignored the size of the piece in this instance.

At the end of the mathematics methods course, the number of correct responses had increased to 28 (68.3%) prospective elementary teachers with a concomitant reduction in the number of incorrect responses to 13 prospective elementary teachers (31.7%). This change was a consequence of 13 (31.7%) prospective elementary teachers moving from incorrectly responding at the beginning of the course to correctly responding at the end of the course, but with four prospective elementary teachers moving from a correct response to an incorrect response at the end of the course (see Table 4). An exact McNemar test determined that the difference in proportion of prospective elementary

Table 4
Contingency Table for Which Is Largest? (n = 41)

Pre-Response	Post-Response: Incorrect *n* (%)	Post-Response: Correct *n* (%)
Incorrect	9 (22.0)	13 (31.7)
Correct	4 (9.8)	15 (36.6)

teachers correctly responding pre- and post-course was statistically significant, $p = .049$. Thus, this methods course did have a positive impact on prospective elementary teachers' ability to identify which fraction was largest from a group of fractions. However, it is concerning that that about one-third of the prospective teachers still could not identify the largest fraction among the three presented at the end of their elementary education coursework.

Discussion and Conclusions

This study revealed that this group of prospective elementary teachers brought with them to their mathematics methods courses an incredibly limited understanding of fractions. Although these prospective teachers had many years of school mathematics, including four college-level mathematics courses and one primary-level mathematics methods course, their reasoning about basic fraction concepts was quite shallow and based heavily on misconceptions they had previously developed. Their understanding of fractions as part of a whole dominated all other interpretations of fractions and remained resilient despite direct efforts in this course to expand and deepen their understanding to include other interpretations and meanings. This is incredibly concerning given that Lamon (2012) suggests that a part-whole understanding of fractions is not sufficient. She further suggests that limiting instruction of fractions to an emphasis on part-whole relationships will leave students with a very limited understanding of fractions. These findings support calls for teachers themselves to have a developed understanding of the multiple interpretations and meanings of fractions (e.g., CBMS, 2001, 2012; NCTM, 1991). This research also supports the idea that helping prospective elementary teachers develop deep and flexible understandings of important mathematics concepts is difficult and challenging and should include dealing directly with their misconceptions and limited understandings. It is important that experiences in their content courses and methods courses not mimic those that have helped create their misconceptions and limited understandings; the experience should be varied and provide prospective teachers opportunities to explore and develop a deep, multifaceted understandings of fractions.

Zhang, Clements, and Ellerton (2015) noted the dominate use of area-model representations in the teaching and learning of fraction concepts. Further, finding of this study support the concern that elementary teachers may be unable to help their students represent mathematical ideas using a variety of representations or to make connections across those representations because they lack the content knowledge themselves. Additionally, research shows that elementary teachers may not able to think about the mathematical ideas in more than one-way (Ball & Bass, 2000). The implications of this study suggest that experiences for students at all levels need to move past a part-whole interpretation and particularly past an area model for understanding fractions. These concerns are complicated by the fact that research on textbook content shows that textbooks lack coherence in their representation and presentation of fraction concepts (Cady, Hodges, & Collins, 2015) and that textbook content focuses predominantly on part-whole interpretations (Lamon, 2001). Given the fact that teachers who are often uncomfortable with the content they are teaching depend heavily on guidance for examples and direction from textbooks, elementary teachers are not likely to encounter much in their teaching to challenge and expand their understanding of fractions, which will continue the cycle of students not developing a deep understanding of fractions. Flores, Samson, and Yanik (2006) suggest that students who have time to work with a variety of interpretations of fractions other than just the part-whole model "will develop a more structured and usable knowledge base" (p. 34) and the results from this study indicate that the same experiences are needed for prospective teachers. Thus, there is a need for continued robust experiences with fractions for prospective teachers, but also for professional development for elementary and middle level teachers to deepen their understanding of fractions and improve their pedagogical practices related to teaching fraction concepts. The findings of this study are in concert with the CBMS' (2012) calls for change in how teachers of mathematics are prepared. They suggest:

> A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics. It is also not enough for teachers just to study mathematics that is more advanced than the

> mathematics they will teach. Importantly, mathematics courses and professional development for elementary teachers should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach. (p. 23)

As mathematics teacher educators, we must recognize the challenge we face regarding improving our prospective elementary teachers' content knowledge through their study of the mathematics they will teach. The mathematics content in any methods course simply provides the context for prospective elementary teachers to experience and discuss research based pedagogical strategies for teaching. As instructors of methods courses, our goal is to help our prospective teachers develop effective teaching methods while also deepening and extending their often fragile understanding of the mathematics content and concepts used as the context for the pedagogic strategies. Since the focus of methods courses is not to teach content, it is important that in the mathematics content courses for prospective teachers effective, research-based pedagogical strategies are utilized with a focus on conceptual understanding rather than procedural fluency. In the case of fraction understanding, various models and interpretations of fractions should be emphasized and used in varied problem-based contexts in the content courses.

The results of this study suggest that the mathematics content and methods courses designed for prospective elementary teachers need to explicitly involve them in making connections within and among multiple representations and meanings of fractions and operations with fractions and whole numbers (Zimmerman, 2014). Tsao (2012) further suggests a need to raise the awareness of the importance of number sense in the mathematical development of prospective teachers and for prospective teachers recognize the need themselves to develop their own fraction sense. It is apparent that reliance on their prior mathematics and simply having prospective elementary teachers study advanced mathematics at college level is not sufficient for helping them understand mathematics with depth nor overcome their preconceived misconceptions. Hill, Rowan, and Ball (2005) have pointed to the need for new policies for improving teacher's mathematical knowledge. Teachers need a good understanding of mathematics content so they can navigate the complex endeavor of helping their students understand the mathematics they are teaching (Ball & Bass, 2000). Hence, it is important that mathematicians and mathematics teacher educators work closely together to examine the content and instructional strategies employed in content courses and collaborate to design the courses for future elementary teachers. Prospective elementary teachers need to develop a deep conceptual understanding of fractions, which should include learning opportunities that involve a variety of tools such as fraction tiles and other manipulatives to explore fraction concepts. Otherwise, prospective elementary teachers' limited understandings and misconceptions will remain resilient and likely be passed along to their future students.

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Influence of a Mathematics Teachers' Circle on Elementary Teachers' Use of Problem Solving

Mary L. Garner
Kennesaw State University

Virginia Watson
Kennesaw State University

Beth Rogers
Kennesaw State University

Catherine Head
Kennesaw State University

Math teachers' circles are a form of professional development that is recommended by the Conference Board of the Mathematical Sciences in their publication Mathematical Education of Teachers II (2012). However, little research has been published on how effective math teachers' circles are in advancing the mathematical knowledge of teachers and influencing their actual teaching practice. This study reports how 25 elementary teachers from schools with minority and economically disadvantaged populations responded to a math circle experience. A pretest and posttest showed a significant increase in problem-solving ability. Through analysis of journals, all but 4 of the 25 teachers indicated that participation in the math circle increased their understanding of problem solving and advanced their use of problem solving in the classroom.

A mathematics teachers' circle is a network of college mathematics professors and K–12 mathematics teachers who meet regularly to solve problems in a collaborative environment. The circle is a professional support network in which teachers can enrich their problem-solving skills, explore new areas of mathematics, tackle interesting problems, and share ideas about pedagogy. The first mathematics teachers' circle was started at the American Institute of Mathematics (AIM) in 2006 and targeted middle grades teachers. According to AIM (2016), the idea for a teachers' circle came from a middle school teacher who regularly attended meetings of the San Jose Math Circle with her students. According to the AIM (2016) website:

> Math Teachers' Circles are well aligned with the Common Core State Standards (CCSS) for Mathematical Practice and are among only a handful of professional development models recommended by the Conference Board of the Mathematical Sciences *Mathematical Education of Teachers II* (2012) (CBMS MET II) document. (http://www.mathteacherscircle.org/about/results/ Fast Facts section, para. 2).

Although the emphasis has been on math circles for middle-grades and high school teachers, the *Mathematical Education of Teachers II* states "the term 'teacher of mathematics' includes early childhood and elementary-level generalist teachers as well as middle grades and high school teachers" (p. 17). The authors also state

> Several points about the CCSS Standards for Mathematical Practice bear emphasizing. First, although those standards were written for K–12 students, they apply to all who do mathematics, including elementary teachers. Second, the features of mathematical practice described in these standards are not intended as separate from mathematical content. Teachers should acquire the types of mathematical expertise described in these standards as they learn mathematics. And finally, engaging in mathematical practice takes time and opportunity, so that coursework and professional development for teachers must be planned with that in mind. Time and opportunity to think about, discuss, and explain mathematical ideas are essential for learning to treat mathematics as a sense-making enterprise. (p. 24)

The eight Common Core State Standards (CCSS) for Mathematical Practice state that students should make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. All activities are hallmarks of effective math teachers' circles. Furthermore, the CCSS expand on the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) original call to emphasize the role of problem solving in the mathematics classroom. Problem solving, along with the other process standards published by the NCTM, has been recommended as the vehicle by which mathematics should be taught. Cai and Lester (2010) reviewed the extensive research on problem solving and concluded that

making true problem solving an integral part of the mathematics classroom develops high-order thinking skills and produces positive attitudes toward mathematics.

There is a growing body of literature designed to help elementary teachers, in particular, to engage their students in problem solving to learn mathematics (Foote, Earnest, Mukhopadhyay, & Curcio, 2014; Lester, 2003; O'Donnell, 2006; Schwartz & Curcio, 2013; Wilburne, 2006). However, according to Sakshaug and Wohlhuter (2010):

> Teaching mathematics through problem solving is a challenge for teachers who learned mathematics by doing exercises. How do teachers develop their own problem solving abilities as well as their abilities to teach mathematics through problem solving? (p. 399)

Furthermore, the authors state:

> Due to improvements in the pre-K–12 mathematics curriculum, teachers are now asked to teach content they did not learn in elementary school and to use processes they did not experience as learners. (p. 399)

In this study, we set out to engage elementary teachers in a math circle and examine how that experience influenced their understanding of problem solving and their use of problem solving in their classrooms. Specifically, our goal was to answer the following research questions:

1. How does participation in a math teachers' circle influence participants' use of problem solving in their teaching?
2. Does participation in a math teachers' circle improve participants' ability to engage in problem solving?
3. Does participation in a math teachers' circle change teachers' attitudes toward mathematics and their identification of themselves as mathematicians?

History of Math Circles

Math circles were originally developed for students. The initiative began in Bulgaria approximately 100 years ago as a way of engaging pre-college students in challenging mathematics. Students and teachers met after school in a relaxed setting and worked on selected problems or discussed specific mathematics topics (Fernandes, Koehler, & Reiter, 2011; Stankova & Rike, 2000; Vandervelde, 2009). The movement eventually spread to the United States. In the fall of 1994, Robert Kaplan, Ellen Kaplan, and Tomas Guillermo founded The Math Circle in Boston. Then in 1998 the Mathematical Science Research Institute, through support from the University of California-Berkeley, began the Berkeley Math Circle. Success of these two groups spawned other groups to form as a means for students to experience and enjoy mathematics. The environments and routines differed across circles but the main theme of extending interest in mathematics was paramount for all groups. According to Stankova and Rike (2000): "Math circles vary in their organization, styles of sessions, and goals. But they all have one thing in common: to inspire in students an understanding of and a lifelong love for mathematics" (p. ix).

Math circles are now beginning to be viewed as a way of also engaging teachers with mathematics. There are approximately 100 math teachers' circles in 38 states, but few of them are designed specifically for elementary teachers. Since 2009, the authors have been running a series of math circles for elementary, middle grades, and secondary teachers.

Literature Review

Research on math circles is sparse, particularly involving elementary teachers. Fernandes et al. (2011) wrote about a middle grades math teachers' circle in which the teachers reported that they learned some mathematics and had a better understanding of how students feel when they are challenged by a mathematical idea or problem. In a study by Marle, Decker, and Khaliqi (2012), participation in math teachers' circles increased teachers' use of inquiry-based practices. White and Donaldson (2011) found that teachers participating in math teachers' circles reported increased enthusiasm for mathematics and started to see themselves as mathematicians. White, Donaldson, Hodge, and Ruff (2013) reported increased mathematical knowledge for teaching. All of these studies focused on middle-grades mathematics teachers.

A mathematics teachers' circle can be seen as a professional learning community. Stoll, Bolam, McMahon, Wallace, and Thomas (2006) provided a general definition of professional learning communities as "a group of people sharing and critically interrogating their practice in an ongoing, reflective, collaborative, inclusive, learning-oriented, growth-promoting way" (p. 223) and presented the evidence for the link between teacher participation in learning communities and enhanced student outcomes. Based on a comprehensive review of the literature, they found that the most important components of a successful learning community are shared purpose, sustained interaction, reflective inquiry, trust, and mutual respect, all attributes of a successful math teachers' circle. Webster-Wright (2009) called for a reconceptualization of teacher professional development, stating that "many professional development

practices still focus on delivering content rather than enhancing learning" (p. 702). She cited two decades of research that demonstrate that effective professional development "continues over the long term and is best situated within a community that supports learning" (p. 703). Episodic workshops in which information is delivered in a didactic manner, with the premise that the professional is deficient and in need of direction rather than "a professional engaged in self-directed learning," is not effective professional development, according to Webster-Wright. A math circle is a long-term community that promotes self-directed learning and collaboration.

One of the goal of math circles is promoting enjoyment of mathematics and an appreciation for problem solving. Thompson (1992) reviewed the literature on what has been called attitudes, beliefs, or conceptions about mathematics and described extensive support for the theory that teachers' attitudes influence classroom practice. Such attitudes have been examined through a wide variety of both qualitative and quantitative methods. Ludlow and Bell (1996) provided an instrument verified psychometrically that measures both attitudes toward mathematics and the teaching of mathematics among preservice teachers.

Research involving the use of problem solving in elementary classrooms is directly relevant to math circles. The term "problem solving" has a history of being problematic (Charles & Silver, 1989; Schoenfeld, 1985). Students and teachers often refer to the exercises that require only application of a learned algorithm as "problems." Problem solving is defined by the NCTM as the "engaging in a task for which the solution method is not known in advance" (NCTM, 2000). Cai and Lester (2010) said problem solving involves:

> mathematical tasks that have the potential to provide intellectual challenges that can enhance students' mathematical development. Such tasks—that is, problems—can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity. (p. 1)

Cai and Lester (2010) state worthwhile tasks are those that allow different solution strategies and tasks that encourage engagement and discourse.

Cai (2003) provides a thorough review of the research on problem solving and cited multiple studies that indicate that young children can problem solve, and such children exhibit higher levels of understanding of the mathematics without sacrificing numerical skills. Cai also states that there is little research on how teachers learn to use problem solving in their classrooms. However, Sakshaug and Wohlhuter (2010) provided evidence that teachers can change from a traditional style of teaching to teaching through problem solving. They studied a group of 41 teachers who were taking an elementary mathematics methods graduate course that was designed to help the teachers become better problem solvers and encourage them to teach mathematics through problem solving. The group engaged with problems appropriate for elementary students and then spent time analyzing the professor's approach to the lesson and anticipating their own students' response to the problems. Through an analysis of journals, the researchers found that teachers' success in the classroom was related to their comfort with the mathematical ideas, a finding echoed by Kersaint and Chappell (2001). Teachers' success was also related to their understanding of what problem solving is and beliefs about problem solving. Teachers' behavior varied with the specific problem being used; depending on the problem, some turned a problem-solving activity into an exercise or made group work optional. A teacher could take on the role of facilitator with one problem, but be very directive with another problem. Overall, the researchers concluded that elementary teachers can benefit from such a short-term professional development opportunity, in learning how and why problem solving can be used in their classrooms.

Theoretical Framework

For the purposes of this study, we created a definition of problem solving that would include many of the features described by Cai and Lester (2010) and consistent with the principles put forward by the NCTM. We define problem solving is a process in which a person exhibits certain behavior in response to a particular type of mathematical question. The mathematical question is one for which the person has NO prescribed solution method and for which there may be multiple solutions; the question does not have to be of any advanced content and is not necessarily a "word problem" or a real-life application of mathematics. The person's response to the question is characterized by:

- Use of a variety of strategies to explore the problem, including solve a simpler problem, consider an analogous problem, try special cases, draw a graph, generate a table of values, and use technology.
- Persistence and patience in seeking a solution, including a willingness to evaluate progress and change course if necessary, and a sense of playfulness and experimentation.

▪ Appreciation of the need to fully understand the problem and constraints.

▪ Development of conjectures and questions inspired by the problem–for example, conjectures about the form and meaning of the solution or the most appropriate solution pathway.

• A willingness to collaborate with others to discuss and compare strategies and solutions including appreciation of the possibility of different approaches.

• Awareness of the need for a convincing argument for the correctness and adequacy of the solution.

Furthermore, the role of the college professor in the math circle is envisioned as one of facilitator, promoting problem solving as described above, and creating an environment for collaborative, reflective, self-directed learning, consistent with the research on successful professional learning communities and the CCSS Standards for Mathematical Practice.

Methodology

Participants

In 2009, a partnership was formed between a regional university and seven high-needs schools from one near-by district, under a Teacher Quality Partnership (TQP) Grant. In 2011, the directors of the TQP Grant allocated funds to provide math teachers' circles for the teachers in the seven schools, including an elementary math teachers' circle for the five elementary schools. A description of the five elementary schools is shown in Table 1. Note that all five schools served populations of students who were from minority groups and economically disadvantaged. The stated purpose of the math circle was to enhance the problem-solving skills and the enjoyment of math for 30 elementary teachers, raise their awareness of the frontiers of mathematics, and deepen their content knowledge. In addition, through the math circle, we sought to provide an opportunity for participants to experience a student-centered environment which emphasized problem solving, reasoning and proof, communicating about mathematics, using multiple representations, and seeking connections between topics in mathematics and outside mathematics. Teachers were paid a stipend of $1000 and were provided $500 in funds for supplies, for their participation in a three-day immersion workshop and three of the four two-hour follow-up sessions.

Math Circle Meetings

The elementary circle began with a three-day "immersion workshop" in which participants immersed themselves in problem solving and got to know and trust each other as a community. The immersion workshop took place during the district's winter break, for eight hours per day. One mathematician or mathematics educator would lead the group in one or a series of problems each afternoon and each morning of the workshop. After the participants had time to work on the problem and discuss the problem in their groups, the leader would bring all the teachers together to discuss methods and possible solutions. At the end of the morning or afternoon session, one of the mathematics educators would lead the group in a discussion of what they could take back to their classroom from the session.

The follow-up sessions were each two hours long and held after school hours at one of the five schools. As in the morning and afternoon sessions in the immersion workshop, one mathematician or mathematics educator would lead the group in one or a series of problems and then lead a discussion of what the participants could take back to their classroom.

Data Collection and Instruments

Both quantitative and qualitative data were collected through three means: journals, an attitude survey, and pretests and posttests.

Journals. On the first day of the immersion workshop, before any math circle activities, the participants were asked to respond to the "pre-workshop journaling" prompt shown in Appendix A. At the end of each day of the immersion workshop and each of the follow-up sessions, participants were also asked to respond to the "end-of-session journaling" prompts shown in Appendix A. In addition, over five weeks after the immersion workshop, participants were asked to respond to the "weekly

Table 1
A Description of the Five Elementary Schools Where Teachers in the Math Circle Taught

School	Economically Disadvantaged (%)	Asian (%)	Black (%)	Hispanic (%)	Multi-racial (%)	White (%)	SPED (%)	ESOL (%)
1	73	4	45	36	4	11	11	24
2	93	0	17	73	3	7	11	51
3	68	2	51	33	6	9	11	27
4	86	2	17	71	2	7	17	52
5	83	4	38	46	4	7	10	36

journaling" questions also shown in Appendix A and send their responses electronically.

Attitude surveys. The survey in Appendix B is a modified version of Ludlow and Bell's (1996) Attitudes Towards Mathematics and its Teaching (ATMAT) survey. The original survey is a six-point Likert scale that includes 29 items and that was tested on prospective elementary teachers and shown to be valid and reliable, reflecting one underlying construct. The survey was modified to include 20 of the original 29 questions. As shown in Appendix B, a few of the 20 items were modified to address practicing teachers rather than prospective teachers; in addition, items 23 and 24 were one item on the original survey. Items 3, 8, 16, 18, 26, and 27 are items that did not appear on the original ATMAT survey. Items 16 and 26 were added to address specifically the use of problem solving as a teaching tool. Items 8, 18, and 27 were added to reflect math circle goals of engaging participants as mathematicians and exposing them to mathematical research. Item 3 was added to replace questions about prospective teachers' thoughts about tutoring and teaching elementary students.

Pretests and posttests. The pretest and posttest questions shown in Appendix C were taken from problems posted by math teachers' circle leaders. Choosing place value problems ensured that the problems would be accessible to all elementary teachers. The problems chosen can be found at multiple sites; one source of questions on place value is: http://www.mathteacherscircle.org/assets/session-materials/HReiterPlaceValueProblems.pdf. Instructions with the pretest and posttest questions were as follows:

> Below is a problem to work on. You may not be able to solve it in the given amount of time. Instead, we are interested in how you approach the problem, what strategies you try in the process of trying to solve it, how you're thinking about the problem. We'd like a "running commentary" as you investigate the problem. Please explain as completely as possible each approach you take and every observation you make (as you work on the problem) even if you later find out the approach is not a good one or your observations are not correct.

Data Analysis

Journals. Each of the four researchers independently examined the pre-workshop journaling and generated codes to classify statements that indicated how the teachers viewed the nature of problem solving and statements that expressed positive or negative emotional reactions to problem-solving activities. All four researchers met regularly until agreement on the coding was reached. Appendix D shows the codes that emerged, along with an example of a coded statement. A code received a count of one whenever a teacher mentioned the topic at all in response to any of the questions for that day. Except for the codes of NER (negative emotional reaction) and PER (positive emotional reaction), each of the codes involves aspects that the teachers associated with problem solving. Two researchers then independently completed the coding on all journaling and met to resolve any differences in the codes.

Attitude survey. Cronbach's alpha was calculated separately for the set of surveys at the beginning of the study and for the set at the end of the study. A paired sample *t*-test was conducted on the mean score at the beginning of the study and the mean score at the end of the study. Responses to the six new questions were also examined both before and after the study.

Pretests and posttests. Pretests and posttests were scored by two researchers according to the rubric shown in Appendix C. The researchers experimented with several more complex rubrics that assessed more of the process of problem solving, but the teachers did not write about their thinking as requested in the directions. After independently scoring the tests, the researchers met and discussed any discrepancies in the scores.

Results

Journals

The results of coding all journals are shown in Tables 2 and 3. Codes that appeared less than three times in all journaling were excluded.

Initial journaling. Table 2 shows that 29 teachers submitted journals. In talking about how they use problem solving in the classroom more than half of the participants mentioned real-life problems. They also mentioned, in order of highest to lowest, multiple representations, prescribed curriculum, word problems, explaining solutions, non-math problems, step-by-step procedures, and group work.

Many comments focused on these as completely describing the problem-solving experience. For example, one participant focused on word problems as the only type of problem solving: "We do a word problem daily in the classroom. I encourage all of my students to make up their own word problems based on basic problems. . .We keep a word problem journal. . ." Other participants wrote of problem-solving experiences closer to our definition. One participant wrote about persistence in problem solving: "As determined as I am I will keep trying possible solutions until I have come to solution that works." Another

Table 2
Codes from Pre-Journaling and Weekly Journaling

	N	SBS	PC	WP	RLP	NMP	CGW	MR	ND	OE	EP	NP	TS	NER	PER	SS
PRE	29	9	12	11	19	9	8	14	1	1	11	3		1	1	2
3/4 week	22	1	7	2	1	0	8	8	3	2	4	0	11	0	1	2
3/11 week	24	2	1	3	8	0	7	1	1	1	2	1	4	2	2	6
3/18 week	24	0	3	3	4	0	5	4	1	1	1	0	5	2	3	3
3/25 week	16	0	2	3	1	0	5	3	0	0	1	1	0	0	0	0
4/1 week	7	0	0	2	0	0	0	2	0	0	1	0	0	0	0	0
Total*		3	13	13	14	0	25	18	5	4	9	2	20	4	6	11

*Total does not include codes from the pre-journaling.

participant talked about how hard it is to engage students in problem solving: "Engaging students in problem solving can be a daunting task. It's a fine line because you want to challenge the students but not so much so that they shut down." While some participants focused on step-by-step procedures to solve problems another saw this as a potential stumbling block:

> Many times the students have a strategy from their previous teacher using a type of acronym...The students are memorizing a procedure without any understanding of the math concepts...As a result the students can't solve word problems.

Finally, one participant wrote of multi-step word problems as an example of problem solving: "Word problem Wednesday—always a multistep math problem in which the students think/pair/share to discuss possible answers."

As we read the journaling the participants roughly divided into three types (a) Type I, those who exemplified our definition of problem solving from the beginning (11 out of 25); (b) Type II, those whose mind was changed by participation in math circles (10 out of 25); (c) Type III, those who never exhibited an understanding of problem solving as in our definition (4 out of 25). The three types are represented by Sarah for Type I, Corinne and Cynthia for Type II, and Fran for Type III. Sarah's pre-journaling indicated her teaching philosophy is closely aligned with our definition of problem solving. She stated, "I try to get my students to perform a task for the first time without giving them a whole lot of direction or instruction. I like to see how the students approach the problem." Conversely, Corinne, Cynthia, and Fran focused on specific step-by-step processes that should always be used, as well as word problems. Corinne stated:

> We have shown the students how to look for important information in the problem and eliminate anything that isn't relevant to the answer... We try to make the students follow the same steps every time they solve a problem.

Week one journaling. On March 4 the participants wrote the first weekly journal entry. Participation in the math circle seemed to strengthen commitment to using problem solving in the classroom or raise awareness of certain aspects of using problem solving even among the Type I participants, those who made statements consistent with our definition of problem solving. Comments from Sarah include the following:

- As you can see, each day built on the previous day's knowledge and the kids were so engrossed in the problem solving because there was no one right answer that we

Table 3
Codes From the End of Circle Sessions, at the End of Each Day of the Immersion Workshop and at the End of Each Follow-Up Circle Session

Date	N	SBS	PC	WP	RLP	NMP	CGW	MR	ND	OE	EP	NP	TS	NER	PER	SS
2/21	28	0	0	0	0	0	0	6	0	3	2	0	1	1	4	0
2/22	29	0	0	0	0	0	1	1	0	0	1	0	0	3	1	1
2/23	28	0	0	0	0	0	3	1	0	0	2	0	1	3	12	0
3/10	24	0	0	0	0	0	0	5	0	0	0	0	0	9	8	0
3/21	15	1	0	0	0	0	1	1	0	0	1	0	0	1	7	0
3/23	15	0	0	0	0	0	0	3	0	0	0	0	1	4	5	1
4/14	2															
Total		1	0	0	0	0	5	17	0	3	6	0	3	21	37	2

actually sustained accountable math discussions amongst my third graders for nearly an hour and that doesn't include the excitement of sharing their results for 20 minutes after the work time during the lesson wrap-up.

- I haven't really thought about the problems of the workshop so much as the process we went through each day and how I wanted to get my students to have the same types of discussions we had during the workshop in my classroom. If this week is any indication I will be incorporating this approach much more often in my classroom.

Other Type I participants had the following comments:

- Although the degree of problem solving is not as intense as what we encountered, I think the process is fundamentally what I am trying to accomplish.
- . . .but the problems were challenging and made me think about math in a way I haven't in a long time. I would like to adapt some of these problems for our children so they can go through the same experiences we went through.
- I tried to give the students as much independence as possible rather than trying to guide them in solving it "my way." The students came up with many different strategies and really listened to each other when they were presenting.
- What interested me were the dynamics of how groups worked through problems, moving from whole-group conjecture, to individual work, to pairs working together, and then back to whole-group (or some similar combination). I discussed some of these experiences with my students early this past week before we worked on the problems discussed above. If anything, it seemed to validate their own experiences, and may well have encouraged them to be better partners and classmates, putting more into their work and expecting more from their partners and classmates.

The ten Type II participants expressed changes in their teaching as a result of participation in the math circle. Corinne stated:

> I have focused more on the amount of discussion about the problems than the actual math that we did during the workshop. I want to try to get my class to be able to have a lengthy conversation about a problem solving activity and use different strategies as a group to figure it out.

Comments from other teachers who evidenced a new conception of problem solving include the following:

- I have tried to think of ways that I can ignite the fire of problem solving in my students. The workshop brought out my love for math. I feel that we all should strive to be problem solvers.
- For the past few days I have been trying to figure out a way to take some of the problems that we worked on and break them down to use in my classroom.
- It made me open my eyes to a lot of different strategies. I discovered that when I am given problems I tend to use the same strategies, but from working with other people and seeing a variety of different problems it opens my mind to try different strategies.
- I want the students to see the problems that were put in front of me, as a teacher, and asked to solve. I want to share with them the frustrations and strategies that I had to discuss and share with other teachers to be able to solve and work out.
- I brought back from the workshop a deeper understanding of the thought processes I use to problem solve and most of all a greater admiration for other peoples' processes and approaches. I've never been a big fan of group work and tend to use it sparingly in the classroom. I realize now that my students could benefit greatly by learning from each other.

Week 2 journaling. Sarah continued to write about her students' problem-solving activities:

> The students again attacked the problem and worked for an extended period of time trying several configurations and double checking each other's figures to prove or disprove the perimeter.

During this time, Cynthia exhibited a change in attitude toward problem solving, stating:

> I've changed my approach by giving the students more time to solve the problems collaboratively instead of me solving the problems with them. I get them to explain more and discuss the different approaches and strategies to [sic] students used to solve different problems.

Among other Type II participants, problem-solving persistence was mentioned frequently and problem-solving stamina became a goal for some of the classroom activities. Comments include:

- The students were showing great endurance and wanted to continue working on more problems like it.
- I was able to give some problems that really make the kids have to stop and think. It was interesting to see how the kids tried to work it out when the easy rule did not work. Just when I thought they had reached the frustration level, they surprised me and told me they where [sic] not

ready for the answer yet... This was a real good day in my classroom... a little shocking as well...

- I have thought a lot about the type of method we are using in math circle which uses a lot of student thinking about problems and results. Most of my students prefer this type of learning method but I have a handful of students who can barely get concepts much less think beyond them. The whole thinking approach has given me a new concept of differentiation and how I can make that process work better.

Conversely, Fran continued to write about word problems, formulas, and procedures.

On this second week of journaling, teachers also began to comment about the Criterion-Referenced Competency Test (CRCT). This required April standardized testing began to loom in the teachers' minds. One Type II participant who evidenced a change in attitude about problem solving commented that he or she "found it difficult to incorporate the problem solving activities as I would like with my students" because of the preparation for the CRCT.

Week 3 journaling. The third weekly journaling continued with positive comments about using problem solving as a teaching strategy among Type II teachers. One wrote,

- I hope to further the problem solving within the classroom to boost their stamina.... Sometimes, I get to [sic] wrapped up in getting the final answer. I am beginning to enjoy working with problem solving.

- I'm giving more thought to them solving problems than just giving them information. This is something new to them, so they don't particularly like it. It's easier for them to be told, but I'm hoping they will be intrigued by finding the answers themselves. I'm going to do more exploratory learning and try to apply problem solving.

- Students should not just get an answer correct. They should be able to justify and explain their results. My kids have really benefited from this activity.

A participant who evidenced our definition of problem solving from the beginning focused on the frustration that comes with a problem-solving activity:

> This has really tested their math problem (solving) and I can see the frustration on their faces that we have faced with you guys. It has been a good frustration, however, because they look forward to working on the problem every day.

Week 4 and 5 journaling. During the week of March 25th, a few comments were made about using problem solving as a teaching strategy but more and more the teachers were focused on the CRCT. As one person noted, CRCT problems are

> nowhere near as useful and thought provoking as the kind of problems they have worked on throughout the year...with all the testing and tutoring, most of the students had little stamina for challenging questions.

One Type II participant wrote about the importance of discussions when problem solving: "I also realized that they were not only having useful discussions, but they were also teaching each other!" A Type I participant wrote about how problems can be solved in different ways: "In the end, the students understood that even if they solved the problem differently than others, doesn't mean that their way of thinking was wrong, it was just different." On the last week of journaling the focus of comments were about the CRCT and journaling trailed off as the end of the school year approached.

Summary of journaling. Tables 2 and 3 list the codes retrieved from each journaling session except for the categories that had less than three codes over all journaling. In the weekly journaling in which the teachers answered questions about what they were doing in their classrooms, the most frequent codes involved: using problem solving as a teaching strategy, using collaborative group work, and using multiple representations. Note that using problem solving as a teaching strategy never appeared in the initial journaling, and the use of real-life problems received the largest number of codes in the initial journaling. In the journaling that was done at the end of math circle sessions, negative and positive emotional reactions were the most frequent codes, with positive reactions outnumbering negative reactions.

Pretests and Posttests

There were 21 teachers who completed both the pretest and the posttest in problem solving. Results are shown in Table 4. The correlation between scorers was .84 for the pretest and .92 for the posttest. In addition, scorers agreed exactly on 11 of the pretest scores and 14 of the posttest scores, and never differed by more than one score point. The mean of the two scores was used for the final pretest and posttest scores for each participant. The scores for both pretest and posttest ranged from 1 to 5. A paired samples *t*-test indicated scores were significantly higher for the posttest ($M = 3.74$, $SD = 1.37$) than the pretest ($M = 3.05$, $SD = 1.19$), $t(20) = -2.62$, $p = .008$.

Attitude Survey

There were 22 teachers who completed the survey both before and after the series of math circle meetings. Cronbach's alpha for the first set of scores was .94, and

Table 4
Results for the Problem-Solving Pretest and Posttest

	Pretest ($N = 21$)				Posttest ($N = 21$)			
	Mean	*SD*	Max	Min	Mean	*SD*	Max	Min
Scorer 1	3.05	1.28	5	1	3.81	1.47	5	1
Scorer 2	3.05	1.20	5	1	3.67	1.32	5	1
Combined	3.05	1.19	5	1	3.74	1.37	5	1

Note. Questions are shown in Appendix C.

Cronbach's alpha for the final set of scores was .87. The means, standard deviation, maximum, and minimum scores are shown in Table 5 for the initial attitude survey and the final survey. A paired samples *t*-test indicated no significant difference in survey scores at the beginning of the study ($M = 3.80$, $SD = .57$) and the end of the study ($M = 3.76$, $SD = .42$), $t(21) = .29$, $p = .77$. Note that scores on items 26 and 16 did not change and were consistently high. Mean scores on the other new items ranged from 3.18 to 3.82 and changed little from beginning to end of the study.

Discussion

In answer to the first research question about how participation in a math teachers' circle influences participants' use of problem solving in their teaching, the journal data indicated that 10 out of 25 teachers moved from talking about problem solving as word problems, step-by-step procedures, and real-life problems to talking about problems solving as a teaching strategy, about building problem-solving stamina, and stimulating collaborative efforts at problem solving. A particularly striking comment by one of the 10 whose conception of problem solving changed said:

> I was able to give some problems that really make the kids have to stop and think. It was interesting to see how the kids tried to work it out when the easy rule did not work. Just when I thought they had reached the frustration level, they surprised me and told me they where [sic] not ready for the answer yet... This was a real good day in my classroom... a little shocking as well...

Table 5
Attitude Survey Results

	Pretest ($N = 22$)				Posttest ($N = 22$)			
	Mean	*SD*	Max	Min	Mean	*SD*	Max	Min
All 27 items	3.80	.57	4.78	2.56	3.76	.42	4.41	2.85
Items 16 and 26	4.25	.46	5.00	3.50	4.27	.40	5.00	3.50

Note. Items are shown in Appendix B.

However, even Sarah, one of the 11 teachers who evidenced an understanding of problem solving consistent with our definition from the very beginning of the study, seemed to reenergize her commitment to problem solving as evidenced in the following quote:

> I wanted to get my students to have the same types of discussions we had during the workshop in my classroom. If this week is any indication I will be incorporating this approach much more often in my classroom.

The participants' commitment to challenging their students and using problem solving comes in high-needs schools with a large percentage of minority and ESOL students.

In answer to the second research question regarding how participation in math circles influences participants' ability to engage in problem solving, the pretest and posttest showed a significant increase in problem-solving ability. Although the pretest and posttest questions were chosen to be as similar as possible in content and difficulty, the difference could be due to the specific problems. The difference could also possibly be attributed to the relationship built between the leaders of the math circle and the participants. Math circles must build trust among all members, and participants may have been more comfortable engaging in problem solving at the end of the study.

Unfortunately, we did not see attitude changes in the survey in answer to the third research question. This may be due to the very short time of the study. However, in the journaling, there were comments made by participants that imply their attitude toward the use of problem solving in teaching did change. For example, one participant stated:

> I'm giving more thought to them solving problems than just giving them information. This is something new to them, so they don't particularly like it. It's easier for them to be told, but I'm hoping they will be intrigued by finding the answers themselves. I'm going to do more exploratory learning and try to apply problem solving.

The results echo the findings of Sakshaug and Wohlhuter (2010) and Kersaint and Chappell (2001) among elementary teachers. Furthermore, in answer to Sakshaug and Wohlhuter's concern about elementary teachers' lack of experience with a student-centered, problem-solving approach to teaching, many of our teachers responded strongly to their experience in math circles. For example, one participant said, "I've never been a big fan of group

work and tend to use it sparingly in the classroom. I realize now that my students could benefit greatly by learning from each other." Some were surprised at how their students responded to problem-solving situations, such as the one who stated "we actually sustained accountable math discussions amongst my third graders for nearly an hour" and another stated "just when I thought they had reached the frustration level, they surprised me and told me they where [sic] not ready for the answer yet" and expressed shock at this reaction.

Limitations of the study include the absence of demographic information on the teachers, which might have indicated differences in response to the math circle according to gender, grade taught, or years teaching. Another limitation was the short time we had the participants; consequently, we did not see a change in the attitude survey. In addition, they were being paid well for their participation and so were very motivated to attend. We tried to continue the math circle beyond the study and the paid sessions, but had difficulty finding times after school that were convenient. Furthermore, the teachers reacted differently to different problems, becoming very involved in the discussion of some problems and seemingly indifferent to others. Further research is needed to examine how to select problems for the circle sessions, how years of experience or other factors influence participants' response to a circle, how to best recruit elementary teachers to the circle, and the best ways to sustain interest.

A math circle is not designed to deliver content, provide curricular materials, or provide training in certain teaching methods. Instead, the goal is to engage the teachers with mathematics in a way they may have never experienced, engage the teachers in exploring mathematics through problem solving in ways consistent with the CCSS Standards for Mathematical Practice. This study shows that even a relatively short experience with a math circle can influence the teachers' problem-solving skills as well as their teaching of mathematics.

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Authors' Notes

Mary L. Garner, Department of Mathematics, Kennesaw State University; Virginia Watson, Department of Mathematics, Kennesaw State University; Beth Rogers, Department of Mathematics, Kennesaw State University; Catherine Head, Department of Mathematics, Kennesaw State University.

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Mary L. Garner, 4621 Ivygate Circle, Atlanta, GA 30339. Email: mgarner@kennesaw.edu.

Appendix A

Questions for Journals

Pre-workshop journaling:

1. Describe how you have engaged your students in problem solving in your mathematics classes over the past year.
2. Describe a successful problem-solving attempt that you've had personally.
3. Describe a not-so-successful problem-solving attempt.

End of session journaling:

1. Did you make a conjecture today? What was it?
2. What problem-solving experience was successful today?
3. What problem-solving experience was not so successful?

Weekly journaling:

1. How did you incorporate problem solving in your classroom this week?
2. What thoughts have you had on the current problem?

Appendix B

Attitude Survey Questions

1. Mathematics is very interesting to me. (Modified)
2. My mind goes blank, and I am unable to think clearly when doing mathematics.
3. I enjoy teaching mathematics. (New)
4. Mathematics makes me feel uncomfortable. (Modified)
5. Mathematics is a subject which I have always enjoyed studying.
6. Problem solving fascinates me.
7. I have a fear of not being able to do math. (Modified)
8. I think of myself as a mathematician. (New)
9. I feel a definite positive reaction to mathematics; it's enjoyable.
10. It makes me nervous to even think about doing a math problem.
11. If I am confronted with a new mathematical situation, I can cope with it because I have a good background in mathematics.
12. I get flustered if I am confronted with a problem different from the problems I've seen in the past. (Modified)
13. I do not attempt to work a problem without referring to a textbook or other reference. (Modified)
14. I can draw upon a wide variety of mathematical techniques to solve a particular problem.
15. I do not feel that I have a good working knowledge of mathematics. (Modified)
16. Problem solving is an important part of every mathematics class I teach. (New)
17. I believe that if I work long enough on a mathematics problem, I will be able to solve it.
18. I enjoy working on mathematics. (New)
19. I have forgotten many of the mathematical concepts that I learned.
20. I learn mathematics by understanding the underlying principles not by memorizing rules.
21. I feel a sense of insecurity when doing math.
22. If I cannot solve a mathematics problem, at least I know a general method of attacking it.
23. Mathematics problems are a challenge. (Modified)
24. Solving problems provides satisfaction. (Modified)
25. I have more confidence in my ability to teach with mathematics than in my ability to teach other academic subjects.
26. Problem solving is a useful way to learn new mathematics. (New)
27. I have no idea what new mathematics is being created today. (New)

Appendix C

Pretest and Posttest Questions and Rubric

Pretest question: What is the largest 5-digit multiple of 11 that has exactly 3 different digits?

Posttest question: Find the smallest integer multiple of 84 whose decimal representation uses just two digits 6 and 7.

Problem-solving rubric:

Points	Description
5	Correct answer; and systematic, thoughtful method is described; and justification is clear for why it's a correct answer.
4	Correct answer; BUT systematic method unclear or not described and no justification for why it's the correct answer. OR Incorrect answer but close to correct, and a valid, thoughtful, systematic method is described.
3	Some systematic effort shown with persistence but no solution or solution is way off. OR Some systematic effort shown with persistence but approach invalid.
2	Some systematic effort but little persistence; no solution. OR No clear systematic approach but lots of activity.
1	No solution; minimal effort; no systematic method shown; and/or serious mathematical errors.

Appendix D

Codes

Code		Example
Step by step process/procedure	SBS	"We usually demonstrate and go over steps on comprehending problem solving. . ."
Rules and formulas	RF	"I was recently looking at high school math and I couldn't figure out the problems because I couldn't remember the old formulas."
Prescribed curriculum	PC	"Once a month we do a school-wide exemplar that incorporates problem solving. . ."
Word problem	WP	In response to how you engage your students in problem solving: "We do a word problem daily in the classroom."
Real-life problems	RLP	"We use a lot of real-life situations that students can relate to."
Solving non-math problems	NMP	"Putting together a baby crib."
Collaboration/Group work	CGW	"I have engaged students in problem solving in my class by working cooperatively with their peers in groups or partners."
Multiple representations	MR	"The students came up with many different strategies and really listened to each other when they were presenting."
Error analysis	EA	"My students use problem solving in various activities done in class, . . .error analysis, . . ."
Gaming strategies	GS	"Working on a Rubik's cube."
Limited direction	ND	"I try to get my students to perform a task for the first time w/o giving them a whole lot of direction or instruction."
Open ended problems	OE	"Students were also excited to have come up with a different answer from their classmates that was also correct instead of their only being one answer."
Explaining solutions and proof	EP	". . .the kids would come in the next day and explain how they solved the problem . . ."
Creating new problems	PNP	"I had the students make up their own examples."
Making conjectures	MCJ	"The students conjectured . . ."
All problems have solutions	SO	"I know that most problems should have a solution."
Teaching strategy	TS	"The current problem is based on a performance standard that we are currently teaching but because the students are manipulating and testing their own conjectures I feel they are gaining a deeper understanding with more enjoyment for the process."
Negative emotional reaction	NER	"Whenever it takes longer than 20–30 minutes to figure out a problem, I usually skip it or shut down."
Positive emotional reaction	PER	"I felt successful with all of the problems today."
Problem solving involves stamina	SS	"I hope to further the problem solving within the classroom to boost their stamina."

Teachers' Understandings of Realistic Contexts to Capitalize on Students' Prior Knowledge

Gloriana González

University of Illinois at Urbana-Champaign

The theory of realistic mathematics education establishes that framing mathematics problems in realistic contexts can provide opportunities for guided reinvention. Using data from a study group, I examine geometry teachers' perspectives regarding realistic contexts during a lesson study cycle. I ask the following. (a) What are the participants' perspectives regarding realistic contexts that elicit students' prior knowledge? (b) How are the participants' perspectives of realistic contexts related to teachers' instructional obligations? (c) How do the participants draw upon these perspectives when designing a lesson? The participants identified five characteristics that are needed for realistic contexts: providing entry points to mathematics, using "catchy" and "youthful" contexts, selecting personal contexts for the students, using contexts that are not "too fake" or "forced," and connecting to the lesson's mathematical content. These characteristics largely relate to the institutional, interpersonal, and individual obligations with some connections with the disciplinary obligation. The participants considered these characteristics when identifying a realistic context for a problem-based lesson. The context promoted mathematical connections. In addition, the teachers varied the context to increase the relevance for their students. The study has implications for supporting teachers' implementation of problem-based instruction by attending to teachers' perspectives regarding the obligations shaping their work.

Recent calls for the equitable participation of all students in mathematics classrooms have emphasized finding realistic contexts in which to situate mathematical problems. Realistic contexts for mathematics problems can increase student engagement, instill in students a perception of mathematics as valuable, and allow students to apply their real-world knowledge to problem solving (Boaler, 2008). In addition, by selecting contexts related to students' backgrounds, students could apply strategies that would make sense in that context (e.g., Drake et al., 2015). A relevant question to ask is how teachers determine what constitutes a realistic context for their students. In problem-based lessons, teachers give students a problem that will help them learn a new concept in the process of solving that problem (Lampert, 2001). In the United States, the implementation of the Common Core State Standards for Mathematics (CCSSM) has promoted the adoption of problem-based curricula (National Governors Association Center for Best Practices [NGAC], 2010). Even though some problem-based textbooks are available (e.g., Dietiker et al., 2006), schools and districts vary in adopting a problem-based mathematics curriculum. Teachers from schools without a problem-based curriculum sometimes craft problem-based lessons to allow students to experience this type of instruction.

I examine teachers' understandings of what constitutes a realistic context in which students can use their prior knowledge. The data come from teacher discussions in a professional development intervention that aimed to increase teacher noticing of students' prior knowledge. Five geometry teachers participated in a study group that involved a lesson study cycle (Fernandez, 2002). The project director (the author) recruited participants by announcing the project in high-need schools which were diverse in terms of ethnicity and social class.[1] During the cycle, the teachers planned a problem-based lesson to elicit students' prior knowledge, taught the lesson, and discussed its implementation. The following research questions guide this study.

1. What are the participants' perspectives regarding the realistic contexts that elicit students' prior knowledge?
2. How are the participants' perspectives regarding realistic contexts related to teachers' instructional obligations?
3. How do the participants draw upon these perspectives when designing the lesson for the lesson study cycle?

My goal is to identify the characteristics of realistic contexts that would maximize students' learning opportunities from the teacher's point of view. The study intends to help unpack the *practical rationality of mathematics teaching* (Herbst & Chazan, 2003) by identifying teachers' decisions and their justifications for these decisions. Understanding teachers' knowledge of teaching and of their students is important for increasing teachers' capabilities when implementing recommendations for mathematics education reform.

Theoretical Framework

Two main perspectives provide the theoretical framework guiding this study. One perspective is the theory of realistic mathematics education (RME). The other is the construct of the practical rationality of mathematics teaching, which attempts to understand teachers' perspectives about teaching actions and their justifications of these perspectives.

Contexts in RME

RME states that using realistic contexts to frame mathematics problems provides opportunities for students to engage in guided reinvention (Freudenthal, 1991). Guided reinvention refers to the process by which students make sense of mathematical ideas. Gravemeijer and Doorman (1999) explain, "The idea is to allow learners to come to regard the knowledge they acquire as their own private knowledge" (p. 116). RME embraces a notion of reality that does not necessarily conform with the use of actual facts. Freudenthal (1991) stated, "I prefer to apply the term 'reality' to that which at a certain stage common sense experiences as real" (p. 17). In Freudenthal's sense, "reality" is a setting that enables students to engage in mathematical activity and reinvent mathematics. Students reinvent mathematics through their work on a problem by developing an understanding of important mathematical concepts that are already known in the discipline of mathematics but are new to them. The contexts that frame problems can refer to real-world situations or to a fantasy world, provided that students perceive these contexts as authentic (van den Heuvel-Panhuizen, 2003).

One of the main assumptions of RME is that mathematics is as an activity. Freudenthal (1971) stated, "It [mathematics] is an activity of solving problems, of looking for problems, but it is also an activity of organizing a subject matter" (pp. 413–414). This activity is called "mathematizing," and it is analogous to other recognized mathematical activities, such as axiomatizing or formalizing. According to Freudenthal (1991), mathematizing should stem from students' own experiences. Student-relevant contexts can provide opportunities to use student experiences as a starting point for conceptualizing mathematical ideas.

When working on mathematical problems that are situated in a specific context, students need to rely upon their prior knowledge of that context. Students may approach mathematical problems differently according to their understanding of contexts, and the choice of contexts tends to privilege some students (Lubienski & Crockett, 2007; Tate, 1994). Work regarding students' funds of knowledge (González, Andrade, Civil, & Moll, 2001) and children's multiple mathematical knowledge bases (Drake et al., 2015) emphasize that connections between students' backgrounds and mathematics increase students' opportunities to understand mathematical ideas. By selecting a context relevant to their students, teachers can demonstrate that they value students' contributions and further the students' development of mathematical knowledge. Students, in turn, may be able to use their knowledge of a problem's context for guided reinvention. Therefore, teachers' selection of contexts in which students can use their experiences is important when striving for inclusive mathematics classrooms.

The Practical Rationality of Mathematics Teaching

The practical rationality of mathematics teaching (Herbst & Chazan, 2011) refers to practices that make sense from the teacher's point of view and that facilitate the fulfillment of teaching responsibilities in light of teachers' multiple demands. Teachers and students organize their activities in classrooms according to an implicit didactical contract that connects students, teachers, and the subject matter (Brousseau, 1997). In typical classroom activities for specific courses, teachers and students act under the assumption that following implicit norms will translate into student learning opportunities. Problem-based instruction requires teachers to conceive of opportunities in which they can use students' work on a problem to elicit mathematical ideas. In geometry instruction, teaching with a problem may conflict with typical instruction (Herbst, 2006). Specifically, a geometry teacher may be challenged in supporting students' development of mathematical conjectures based upon students' work on a problem because typical patterns of interaction do not require students to develop new knowledge by making conjectures. Understanding teachers' perspectives regarding realistic contexts is a step toward making problem-based instruction viable in the classroom.

Recent work has established a connection between the practical rationality of teaching and teachers' obligations by identifying four conditions that shape teachers' work (Bieda, Sela, & Chazan, 2015; Herbst & Balacheff, 2009; Herbst & Chazan, 2011). The first condition is the relationship between school mathematics and mathematics as a discipline. The second condition is the institutional obligation of promoting the development of students' mathematical knowledge as stated in the school standards and policies. The third condition is the obligation of teachers to attend to students' individual differences. The final condition is the interpersonal obligations between teachers and students when interacting in school.

Professional Development Model

The data for this study come from a two-year professional development intervention funded by the

National Science Foundation in the Midwestern region of the United States. As stated earlier, the intervention was intended to increase teacher noticing of students' prior knowledge in problem-based instruction. By teacher noticing, I refer to teachers' anticipation of student thinking and the use of students' ideas during instruction (Sherin & van Es, 2005). The intervention was situated in geometry instruction and supported teacher noticing of three sources of prior knowledge: school mathematics, out of school mathematics, and contexts framing mathematical problems. Teachers typically focus on students' prior knowledge of school mathematical content (Hohensee, 2016). However, the intervention promoted teachers' attention to ways in which students' knowledge and experiences shape mathematical problem solving.

The teachers participated in monthly study group sessions that occurred after school. The sessions had a duration of three hours and were co-facilitated by a member of the research team (Facilitator 1) and the author (Facilitator 2). The intervention combined lesson study, animation discussions, and video clubs. First, the participants watched animations with examples of problem-based lessons (see Herbst & Chazan, 2011). Animations are instructional stories with cartoon-based characters; they were shown as slide-shows with a voice-over. Then, the participants planned a research lesson. All the participants taught the lesson and the research team videotaped the lessons. Finally, the participants engaged in video clubs to analyze students' mathematical ideas (Sherin & van Es, 2005). After the first lesson study cycle, the participants revised the lesson and implemented the revised version during the second year.

Participants and Data Sources

Five experienced geometry teachers with 4 to over 20 years of experience teaching mathematics participated in the study group during the two-year program: Alexa, Clara, Erin, Gian, and Madeline (all pseudonyms). The primary data sources for this study are the study group discussions during the first year of the professional development intervention. In particular, comments about the problems' contexts surfaced at three different moments in the lesson study cycle: (a) when discussing animations prior to designing the research lesson, (b) when planning the research lesson, and (c) when revising the research lesson. Ten study group sessions took place during the first year. This article focuses on a lesson about perpendicular bisector properties.[2] In lesson study, the teachers collectively plan a research lesson to examine hypotheses about student thinking in relation to learning goals. In this case, the research lesson was an instrument designed to examine how to elicit and use their students' prior knowledge to teach the properties of a perpendicular bisector. The research team predetermined the concept of the perpendicular bisector by considering the goals stated in the CCSSM (NGAC, 2010). The research team also identified the perpendicular bisector as a concept that is amenable for creating problems situated in various contexts and with multiple connections to other concepts.

The research team produced animations depicting examples of problem-based lessons to promote connections with students' prior knowledge (González & DeJarnette, 2017). All the problems in the animations were situated in a realistic context and were based upon our observations of geometry students' solutions in separate studies. The animations illustrated two different problems to teach the properties of perpendicular bisectors. In the "pottery problem," students identify the perpendicular bisector as the line of reflection (DeJarnette, González, Deal, & Rosado Lausell, 2016). The problem is situated in the context of Hopi and Pueblo pottery and support students' discovery of the symmetry line as the perpendicular bisector of the segments that connect corresponding points in the pre-image and the image. In the "grocery store problem," students have to recognize the perpendicular bisector as a set of points equidistant from two given points (DeJarnette & González, 2016). Students find the best location for a new grocery store by using a map with points representing the locations of existing stores. Ultimately, the teacher-character wants students to construct the perpendicular bisector of the segments connecting the three pairs of chosen points to find the circumcenter. These two problems highlight different properties of a perpendicular bisector, which are taught in two different curricular units, during the transformations unit (CSSM Standard HSG, CO.A4) and when discussing theorems about lines and angles (CSSM, Standard HSG.CO.C9).

The animation discussions were part of the instructional materials research step and were intended to spur ideas for creating a problem for the research lesson (González & Deal, 2017). In this step, the participants also studied curricular materials, wrote sample problems, and selected one problem for the research lesson. The animations demonstrated alternative scenarios during the launch, exploration, and summary of a problem-based lesson.

Methods

Members of the research team initially parsed the sessions into intervals, a unit of analysis based upon changes in the activity structure during the session (Herbst,

Nachlieli, & Chazan, 2011). For example, in one interval, the participants may be watching an animation, while in another one, they may be discussing ideas for the lesson. The average length of the intervals for Year 1 of the study group was 3:26 minutes, which provided a more manageable unit of analysis than the entire session. I selected intervals in which participants discussed issues pertaining to the context of a problem during the animation discussions or the lesson study steps.[3] Since the sessions integrated various activities, multiple opportunities arose for participants to consider problem contexts in the same session. The participants discussed the problems' contexts in a total of 24 intervals (2 intervals in session 1, 7 intervals in session 2, 3 intervals in session 8, 5 in session 9, and 7 intervals in session 10). In 10 of those intervals, the participants were revising the research lesson, and in 12 of those intervals, the participants were discussing animations. One interval pertained to the instructional materials research (session 1), and another one related to a discussion of the year's accomplishments (session 10).

The selected intervals were fully transcribed. Using those transcripts, I identified specific comments about the contexts of the problems in relation to the animations shown, the lessons that they usually taught, and the research lesson. I analyzed the transcript comments highlighting participants' statements. Then, I made a list of themes that surfaced when describing the characteristics of the contexts using open coding. I condensed some of the characteristics that were repeated across intervals because excerpts from the transcripts illustrated similar positions. Since the themes surfaced during the study group discussions, I assumed that the participants developed shared knowledge about the characteristics of realistic contexts. These characteristics could have contradicted each other, and I take them as different perspectives.[4] To answer the second research question, I examined the justifications for the context characteristics and mapped them with the instructional obligations. To answer the third research question, I listed the contexts selected and identified the reasons that they stated for changing the context.

Findings and Discussion

Characteristics of Realistic Contexts

The participants described five characteristics that realistic contexts needed: (a) the provision of entry points to the mathematics, (b) "catchy" and "youthful" contexts, (c) contexts that are personal to the students, (d) contexts that are not "too fake" or "forced," and (e) contexts connected to the lesson's mathematical content. Most of these characteristics surfaced during the discussion of the grocery store problem (11 intervals). By contrast, the pottery problem did not elicit much discussion about the context (1 interval). One possible reason for this disparity is that the participants' discussion of the pottery problem focused on the likelihood of teaching perpendicular bisectors within the transformations unit. The participants stated that teaching the definition of a perpendicular bisector in the transformations unit is atypical. Another possible reason is that the participants considered the context to be unfamiliar to their students. Madeline's first reaction to the animation was, "I was just saying that, like, if I were reading this I would be like, 'Oh Hopi and Pueblo pottery, like, what are you talking about?'" The participants seemed to consider the grocery store problem more viable than the pottery problem.

Providing entry points into the mathematical content of the lesson. The participants established that a problem's context could provide opportunities for students to become engaged with the mathematical content of the lesson. The participants identified that the ability of all students to choose to contribute to the initial discussion as one positive aspect of the grocery store problem. Madeline voiced this perspective in the following quote.[5]

> *Madeline*:I think it was good because, I mean, at one point almost every student, or at [least] it sounded as though every student, had some kind of input. So, it was—he [the teacher-character] was engaging them in their prior knowledge to get everybody kind of talking about what are some different things that, that we need here or that we could think about.

Later in the session, Madeline elaborated on the opportunities to engage students by using this context, especially for those whom she labeled "lower performing students."

> *Madeline*:. . .And what I like about it is it does for your, maybe, your lower performing students; it gives them an opportunity to be someone who could be vocal in the classroom without it being math, and they are scared of being wrong or you know they've been wrong before when they talked about math out loud and are embarrassed by it. [. . . .] So, it's like they get a chance to actually be engaged in the material before it starts getting super mathy and they kind of shut down a little bit. So, it's like when you start with that whole, like, "we are going to talk about perpendicular bisectors," those kids are like automatically [shutdown noise].

This quotation reveals a contrast between starting the lesson with references to the target mathematical concept (i.e., perpendicular bisector) or the problem's context (i.e., grocery stores). If a teacher were to start the lesson by mentioning perpendicular bisectors, students would disengage. However, talking about grocery stores can enable students' participation, especially when considering those students who otherwise would feel excluded. Therefore, a context that engages students is important in giving students an opportunity to participate in mathematics classrooms. This characteristic considers interpersonal and institutional obligations. By creating a space in which students can voice their opinions about a context, a teacher can engage students who would otherwise be disempowered in mathematics classrooms, feeling that they do not have anything to contribute. A teacher can also use student engagement to introduce the lesson's mathematical ideas and thus fulfill the institutional obligation of teaching students the specific content indicated in the curriculum. From the participants' perspective, selecting a realistic context for their students is a means of achieving inclusive classrooms.

Something "catchy" and "youthful." Another characteristic surfaced in the discussion of an animation about launching the grocery store problem, which we called "Launch 3." In that launch, the teacher-character tells the class to imagine that they are on an urban planning team and asks them to name factors affecting the selection of the location of a grocery store. The students mention various factors including population density and sales tax. Then, the teacher-character asks the students to be the mathematicians on the team. The participants questioned whether finding the location for a grocery store would be appealing for their students. For example, Gian said, "Our kids are going to be thinking, 'Can I get to this grocery store and back before the bell rings, you know,. during lunch?'" The participants identified the teacher-character's emphasis on finding a convenient location for pedestrians as one potentially useful aspect of the launch since many geometry students do not have a driver's license yet.

Identifying a context that would promote students' active participation prompted ideas about changing the problem. Gian proposed changing the problem to finding the site of a fast food restaurant. Madeline stated that the place to be located should be "catchy" and "youthful."

> *Madeline*: And grocery stores they can relate to. But, at the same time, like, yeah, what Gian is saying, like, that makes—I mean and not necessarily McDonald's—I feel like we got a lot of those, but, but you know, but it is something, something more catchy or youthful or like—
> *Gian*: They have more buy-in and they're gonna have more—
> *Madeline*: More—right.
> *Gian*: If you're trying to initiate the conversation, they're gonna be more, "I like McDonald's!" You know, that thing will come up versus, "I like Taco Bell." "Well, I eat there." "What if McDonald's moves in with Taco Bell?" Or whatever, you know. I think you'll get those spin-offs, but with a grocery store, it's not as interesting. They don't have a favorite grocery store.

The discussion of the grocery store problem enabled the participants to refine the idea of the characteristics of contexts that would be the most appealing to their students. Following Gian's comments in which he established that finding a location for a fast food restaurant would be more appealing to students than a grocery store, the participants referred to another problem, "the hospital problem." Alexa and Madeline introduced the hospital problem during the instructional materials research step of lesson study in the first study group session. In that session, the participants shared their typical lessons for teaching perpendicular bisectors (González & Deal, 2017). The hospital problem involves finding the best location for a hospital considering three cities in neighboring states (Zbiek & Foletta, 2006). Madeline and Alexa, who teach at the same school, used this problem in the past to teach perpendicular bisectors. The discussion of the grocery store problem in the second session provided them an opportunity to revisit the hospital problem and question whether the context was appealing to their students.

> *Madeline*: It's kinda like our hospital problem.
> *Alexa*: Right.
> *Madeline*: When we go to make a hospital and they, they don't care.
> *Alexa*: [Laughs.]
> *Madeline*: "I don't wanna be in the hospital." [Laughs.]
> *Facilitator 2*: Have you done that problem?
> *Madeline*: Yes.
> *Facilitator 2*: You've done that problem with the hospital?
> *Madeline*: We've done that problem, yeah. That's our big, that's what we do with perpendicular bisectors. That's our big, it's kind of like the problem-based learning thing.
> *Alexa*: Yeah, a catch.

Madeline Like, here's the problem, how can we solve it?
Facilitator 2: And, what was the reaction?
Madeline: I mean they like it 'cause it's like a colorful map and it's like a real thing. But I guarantee you that if it was like [towns near their school]. If you use three towns that they knew of, 'cause right now it's like, "New Mexico something." You know.
Alexa: Wyoming.
Madeline: If you use three towns that they knew of, and you said we wanna build like a skateland or we wanna build, you know, something that kids would be more into, like those odyssey places where it's like arcade games and skating and dadada—you know. Then I feel like they'd be like, "Yeah, we need one of those."

Madeline and Alexa stated that students' engagement with the hospital problem was related to the resources for the task (i.e., the colorful map) and the real-world scenario. They used humor to describe students' reaction to the problem ("I don't wanna be in the hospital"). According to the participants, the contexts of the grocery store and the hospital problems are not sufficiently engaging. Analyzing these contexts enabled them to envision alternative sites that would be more appealing. Selecting a context for the problem that is "catchy" and "youthful" seems to be related to the obligation of establishing interpersonal relations with students and teachers' attention to students as individuals. Finding a context that is related to students' interests would increase students' motivation and active participation. A problem's context about finding a new location for a building requires a building that students typically visit or would like to visit. Student investment in the context would seem to make the problem realistic.

A context personal to the students. According to the participants, identifying a context in which students can establish connections with their own experiences is important. This idea surfaced while discussing the fact that many geometry students do not have their drivers' license yet and are therefore pedestrians. The following quote presents another example explaining how this theme surfaced in the discussion of the hospital problem. In the quote, situating a problem that is geographically far away from the students' setting could limit the relevance of the context.

Madeline: And I feel like some of the stuff we do like that you pull straight out of textbooks like, "We're building a hospital between New Mexico and duhduhduh," it's like, it's the right idea, but it's not personal enough for our kids to really like—
Alexa: —get the connection.
Madeline: Hook into it, yeah.

According to the participants, even though some contexts in curricular materials show examples of applying mathematics to real-world scenarios, students perceive these as unfamiliar. The notion that a context should "hook" students into solving the problem seems to guide the selection of a context that is personal for the students. This characteristic seems to be related to the obligation of meeting students' individual needs. Teachers will apparently increase students' motivation by establishing links between mathematical problems and students' experiences.

A context that is not too fake or forced. The participants stated a context can appear to be "too fake" or "forced" when discussing the grocery store problem. The participants showed concern that emphasizing the problem's context could make the problem seem inauthentic.

Alexa: But I think you gotta be careful because I think there is a disadvantage of going too much with the prior knowledge, because then it becomes too fake. And then you lose them that direction too.
Madeline: What do you mean "too fake"?
Alexa: That they know it's just, that you are just making this up for, you know what I mean—
Clara: Inauthentic.
Alexa: Kind of like, yeah, kind of like the hospital thing.
Gian: You were right. Pizza Hut is not going to get a drone. I think that is what you are saying.
Alexa: Well, no I don't mean that. I mean if you're going—
Madeline: You're meaning if it is forced?
Alexa: Yeah.

Alexa stated that a potential difficulty of emphasizing the context of a problem could be that students become aware of its artificial nature. Alexa mentioned the hospital problem as one that students can perceive as fake. Gian mentioned the problem of finding the location for a restaurant that delivers food with drones as another possible example of a fake context. However, Alexa rejected Gian's example. It could be that the problem of delivering food with drones was more authentic to Alexa than the hospital problem. While the reason that the participants characterized the hospital problem's context as fake is unclear, the students apparently must perceive a context as

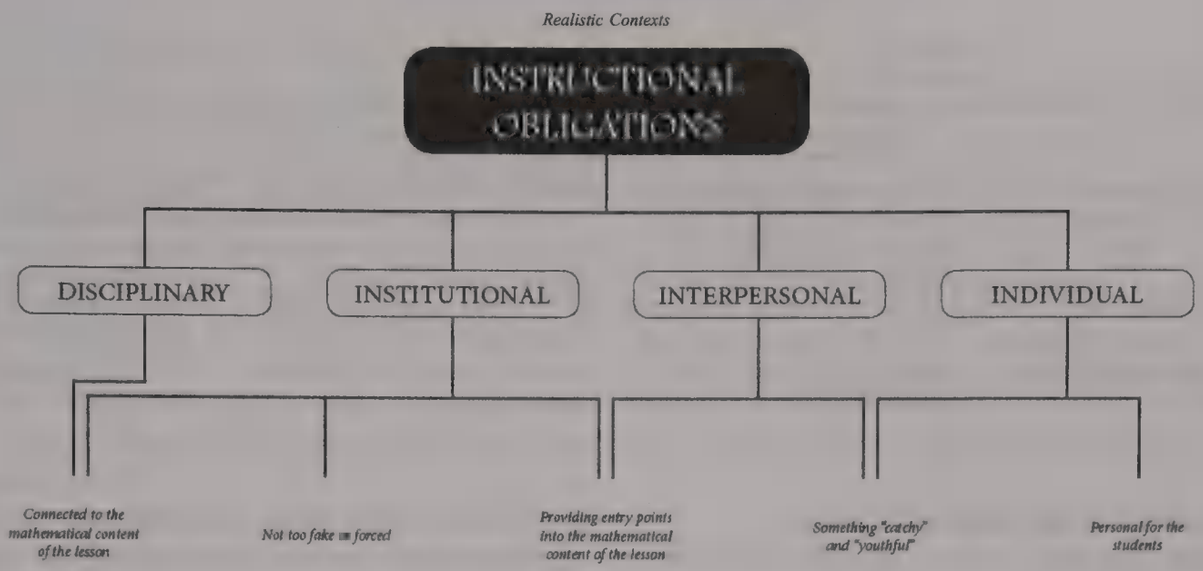

Figure 1. Characteristics of contexts in relation to instructional obligations.

authentic. Alexa said later, "you can use all this prior knowledge, but if it's forced, it doesn't work." Trying to force a context into a particular mathematical situation can jeopardize the opportunity to immerse students into the problem. This characteristic is related to the teacher's institutional obligation. Brousseau (1997) states that teachers are responsible for selecting problems that would motivate students to learn. Teachers' identification of contexts that are not too fake or forced can support the fulfillment of the didactical contract.

Connected to the mathematical content of the lesson. Another characteristic discussed was that a problem's context should be connected to the mathematical content of the lesson. Madeline stated that some interdisciplinary approaches could increase teachers' difficulties in highlighting mathematical ideas. Her comments referred to the implementation of a curriculum that strived to make connections between mathematics and art. Madeline provided an example for a case in which curricular materials do. not establish a meaningful connection. Madeline talked about this example in relation to the discussion regarding contexts that are fake or forced.

> *Madeline*: And I think that happens too, and it is inauthentic when it is where we used to do artful learning. . . . I got to the point where it would become inauthentic. Like, I would be like, "Look at this. It's The Death of Socrates. Let's talk about Socrates loved, you know, processes and thinking, and dadada." "Awesome, let's find volume." [Laughter.]

By contrast, the participants mentioned other lessons with genuine connections between mathematics and art. For example, a typical activity in geometry classrooms for teaching similarity and dilation involves enlarging an art piece. Erin stated that a context that is disconnected from the mathematical content of the lesson could distract students.

> *Erin*: . . . I don't think our kids would say this, but in the video [the animation] they were talking about sales tax, or like, zoning laws, or things like that. Your kids went way off where you're trying not to let them go. So, if the kids are talking about all these things, they are not going to see where you are trying to get them at.

According to the participants, connections should be established between the mathematical content and the context of a problem. When strong connections are lacking, the problem appears contrived. In addition, a context that is not well connected to the mathematical content would make it difficult for students to focus on the relevant mathematical ideas. The participants' perspectives seem to be related to the teachers' obligation to represent the discipline and to the institutional obligation. While some contexts have strong connections to mathematical concepts, others do not, which raises the question of whether novel contexts would provide opportunities to study a concept that is not traditionally seen as connected to that context.

Summary. Figure 1 presents the characteristics of contexts in relation to instructional obligations. Attention to the institutional obligation was most present, followed by individual and interpersonal obligations. Choosing a context that is connected to the mathematical content of the lesson represents the disciplinary obligation. The interpersonal and individual obligations were important for finding realistic contexts that engage students in mathematics learning.

Table 1
Problems for the research lesson

Gian	Alexa and Madeline	Clara	Erin
Finding a location to build an athletic complex between two rival schools (Year 1).	Finding a location to build an after-school center for their high school and nearby middle school (Year 1).	Finding a location to build an athletic complex between two rival schools in a neighboring school district (Year 1).	Finding a location to build a new restaurant of their choice between their school and a neighboring school (Year 1).
Finding the boundary line between two pizza restaurants delivering pizza with drones (Years 1 and 2).	Finding the location to build an after-school center for their school and a local middle school (Year 2).	Finding a location for a YMCA between their school and another school in the district (Year 2).	Finding a location to build a sports complex that is closer to the rival school than to their own school (Year 2).

Contexts Chosen for the Research Lesson

The contexts selected for the research lesson reveal participants' understanding of realistic contexts for their students (Table 1). All the participants designed a lesson about finding locations in a map based upon the grocery store problem. The locations had to be "fair" in relation to two given points. The goal of the "fair location problem" was for students to elaborate on the notion of fairness by finding points that were equidistant to two given points and consequently, on the perpendicular bisector of the segment connecting the two given points.

In Year 1, four participants used a local map, and one participant, Clara, used a map of a neighboring school district. Gian taught two versions of the lesson, one with a local map and another one with a map of a neighboring school district. In Year 2, all the participants used a local map showing examples related to their schools' context. Clara changed in response to her perception that her students were not sufficiently engaged when asked to find a location for a new athletic complex for a neighboring school district. During the lesson revision discussions, Clara commented that she booed after mentioning the rival schools when reading the problem in her class. In Year 2, Clara asked her students to consider where to place a new YMCA[6] between their school and another school in the city. When I asked her about the change, she said, "I think either way, they would have done the problem. But it was less 'boo the other school' like last year, you know? There was less talk about that." She also mentioned that the students asked if the idea of building a new YMCA was true and showed engagement discussing the possibility.

Other changes in Year 2 included variations to the task. For example, Alexa and Madeline realized that the given points were oriented vertically, which facilitated students' discovery of the perpendicular bisector as the horizontal line (Deal & González, 2017). In Year 2, they changed the problem to one about finding a location to build an afterschool center for their school and a middle school in the district; these were not oriented vertically. They anticipated that students would apply their knowledge of the place to their solution.

> *Alexa*: We expect them to say their idea of fair being somewhere in the middle or in-between and that we feel that they're gonna name specific streets or not off specific streets, because they're familiar with it.
> *Madeline*: Near the town.
> *Alexa*: You know, like, "we can't put it here, there's a center there" or whatever.

Other participants also varied the building. In Year 1, Erin asked her students to identify a restaurant that they wanted to build in their town, enabling them to become the cocreators of the context. During Year 2, Erin changed the context to one about locating a sports complex.

> *Erin*: Well 'cause I—when I asked it, I thought that people would have different opinions about the food, and I thought, well, half of the kids didn't hear—haven't heard of some of the locations or something like that. So, I thought kids would tune out. But I thought by changing it to the sports complex, all of the kids know this, and they all probably will have an opinion—or at least, you know, the whole, like, what you were saying, hitting at a rival.

According to Erin, all students would have an investment in finding a location for a new sports complex between her school and a rival school in the same district.

In the first session, Clara shared a problem that she created with her colleagues about finding the delivery line between two fast food restaurants. Gian modified that problem to one about finding a boundary line between two pizza restaurants. He launched the problem by showing a video of drones and by discussing the possibility of delivering pizza with drones. Gian talked

about students' excitement, "I teach geometry four times. By the fourth time, the kids were all like, 'Oh this is the drone thing?'" Gian stated that his students' enthusiasm was atypical. According to Gian, using a local map was engaging. He said, "they [the students] kind of got that they were interested in it 'cause they were all showing—some of them were showing where they lived on the map and things like that, so it had the buy-in from the kids."

Overall, the participants tried to choose "catchy" and "youthful" contexts that provided an entry point to the lesson's mathematical content. The teachers made the context personal for the kids by using a map of their local community, referring to their own schools, and identifying community needs. At the end of the revision process, the participants stated that being able to change the context according to their students' interests was a positive aspect of the lesson.

> *Gian*: I kind of think the strength of this, this activity is the fact that it's the concept and then you can dress it up however you want. You know what I mean? So, it seems like there's some commonality to the activity that everybody is picking a piece of and we're all ending at the same place, but—
> *Madeline*: Making it relevant to our kids.
> *Alexa*: But it doesn't require a high level of math ability.
> *Gian*: True. True. True.
> *Clara*: Right.
> *Erin*: Yeah.
> *Clara*: So you would be able to be involved.
> *Alexa*: It would engage a lot of kids.
> *Gian*: Yeah.

Teachers' changes to the context considered whether the problem was accessible to students, drew upon students' prior experiences and interests, and supported discovery. In that sense, the context was authentic, personal for the students, and promoted equitable participation.

Two Dimensions in Relation to Teachers' Perspectives of Realistic Contexts

Teachers considered two main factors for selecting a realistic context: the mathematical connections and the relevance of the context for the students (Figure 2). Teachers assess the potential connections between a mathematical concept and a problem's context in relation to their knowledge of mathematics and the curriculum. Additionally, teachers' perspectives depend upon their assessment of their students' interests and familiarity with that context.

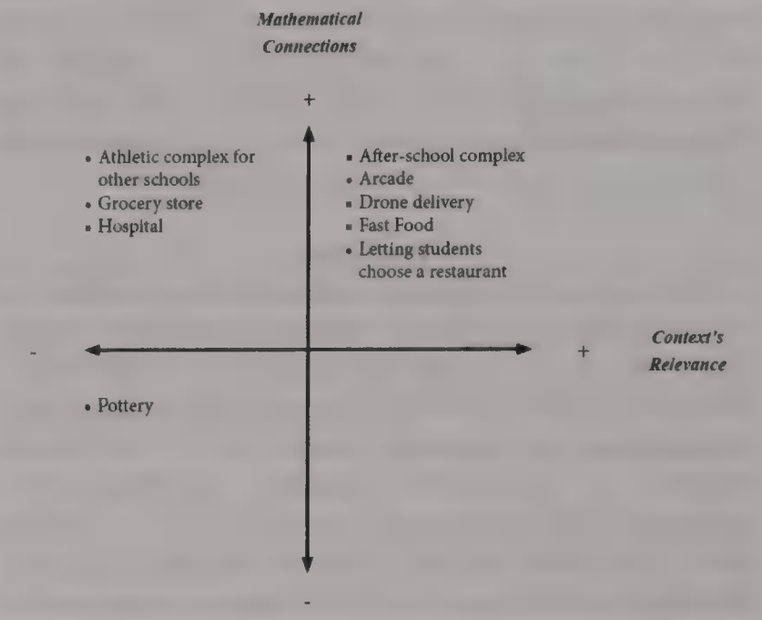

Figure 2. Classification of problems according to two dimensions.

My hypothesis for the participants' rejection of the pottery problem is that in the typical geometry curriculum, the concept of a perpendicular bisector is not usually introduced in the unit on geometric transformations. The participants also did not consider the context of the problem to be appealing to their students. Thus, the pottery problem seemed to limit their opportunities for using context as an entry point to the lesson's mathematical content. A third possible reason is that the context of the pottery problem provides a cultural reference, which is not necessarily connected to the mathematical content of the lesson or specific solution strategies.[7] Two open questions are whether teachers in other geographical areas would find the context of the pottery problem relevant to their students and whether more discussions about opportunities to connect mathematics and culture would shape teachers' perceptions.

By contrast, the grocery store problem can be used when the perpendicular bisector is typically introduced in the geometry curriculum, and it has the potential to make the curricular goals explicit. However, the teachers stated that the grocery store problem, the hospital problem, and finding an athletic complex for a neighboring school district are irrelevant contexts for students. Thus, identifying a problem that is connected to the curriculum is insufficient. Teachers prefer contexts that students perceive as relevant. The teachers appreciated that they could adapt the grocery store problem in relation to students' interests. Contexts that consider the school's geographical location and that are related to places that students visit regularly have the

potential to engage students. Curriculum designers should be asked how they can enable teachers' selection of relevant contexts by using local landmarks, for example. The opportunity to adapt a problem to a local context could make the problem more appealing for students.

Conclusion

Selecting realistic contexts for mathematics is crucial for engaging students in guided reinvention. A realistic context that is relevant to students can enable them to draw upon their prior knowledge in significant ways. When discussing mathematics and reality, Freudenthal (1991) stated, "Reality is historically, culturally, environmentally, individual, and subjectively determined" (p. 17). In this study, the teachers selected a realistic context by adapting the fair location lesson to their classroom community. Teachers' understanding of realistic contexts for their students is important for broadening students' learning opportunities. For students from high-need schools, realistic contexts can provide agency and instill that their expertise is valued.

RME theory has supported the development of mathematics curricula that is grounded in contexts for guided reinvention (van Den Heuvel-Panhuizen, 2003). The identification of realistic contexts is fundamental in allowing students to use their prior knowledge to mathematize. According to Gravemeijer and Doorman (1999), "Well-chosen context problems offer opportunities for the students to develop informal, highly context-specific solution strategies. These informal solution procedures then, may function as foothold inventions, or as catalysts for curtailment, formalization or generalization" (p. 117). While teachers may rely on published problem-based curricula to identify realistic contexts, empowering teachers to become instructional designers can allow them to craft relevant problems for their students according to their local context and students' interests. Promoting equity requires drawing upon teachers' knowledge of the content, their students, and the curriculum to engage all students in mathematics classrooms. Mathematics teachers are best positioned to strengthen the connections between students' knowledge of contexts and mathematical content in the school curriculum.

Considering the small sample size, teachers may have overlooked some characteristics of realistic contexts for problems. In addition, teachers' use of the same adjectives to characterize tasks may imply different meanings and that teachers disagree about their evaluation of tasks in relation to similar criteria (Foster & Inglis, 2017). In that study, the teachers used "applied," "context-based," real-life," and "realistic," to characterize the task context. Therefore, inquiring into teachers' justifications regarding the value of problem contexts is important. Moving forward, it would be important to examine students' perspectives about realistic contexts. Additionally, prior work has identified problem-solving strategies related to students' knowledge of contexts in elementary mathematics (van den Heuvel-Panhuizen, 2005). Examining how high school students draw upon their prior knowledge can support instructional design.

Understanding the practical rationality of mathematics teaching is important when envisioning how to support teachers in enacting reform ideals. Problem-based instruction increases teachers' demands because teachers depend upon students (Cohen, 2011). The animation discussions enabled the teachers to identify criteria for selecting realistic contexts and envision how a problem framed in a particular context would unfold in their classrooms. In reaction to the animations, the teachers rejected and embraced ideas for selecting a context for a lesson about a specific concept. The teachers' participation in a lesson study cycle provided the opportunity to select contexts and revisit their initial choices during the lesson revision process. Through these choices, the teachers capitalized on students' prior knowledge of mathematical content from school, out of school mathematics, and the problem's context. The professional development intervention supported teachers in attending to students' prior knowledge, developing a critical stance toward teaching practices, and establishing shared knowledge about selecting realistic contexts. Developing a vision for enacting problem-based instruction requires increasing opportunities for teachers to both collaborate and share their knowledge of teaching.

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Author's Notes

Keywords: geometry, math/math education, professional development, teachers and teaching, problem solving, learning processes, reform, curriculum teachers' understandings of realistic contexts to capitalize on students' prior knowledge.

[1]According to the criteria established by the National Science Foundation for "high-need schools" in the Robert Noyce program (NSF 11–517), the following criteria must be met: (a) a high percentage of individuals from families with incomes below the poverty line, (b) a high percentage of secondary school teachers not teaching in the content area in which the teachers were trained to teach, or (c) a high teacher turnover rate.

[2]The lesson study cycle was enacted twice to plan two lessons, one teaching the perpendicular bisector and the other teaching dilation. This study only includes the analysis of teachers' selections of contexts pertaining to the perpendicular bisector lesson that took place in the first cycle.

[3]While issues pertaining to the contexts of the problems were relevant to the video clubs, my initial analysis did not reveal that participants considered changes to the problem's context during video clubs discussions.

[4]In their study of the practical rationality of mathematics teaching, Herbst et al. (2011) also consider teachers' comments in group discussions without making claims about individual perspectives.

[5]Transcription conventions show explanatory comments within brackets, elided text using ellipsis, and interruptions with dashes.

[6]"YMCA" stands for "Young Men's Christian Association," which is an organization that owns buildings with athletic and community programs in various cities.

[7]Thanks to an anonymous reviewer for identifying this issue.

Gloriana González, Department of Curriculum and Instruction, University of Illinois at Urbana-Champaign, Champaign, IL 61820. Email: ggonzlz@illinois.edu.

A Statewide Implementation of the Critical Features of Professional Development: Impact on Teacher Outcomes

Carla C. Johnson
Purdue University

Toni Sondergeld
Drexel University

Janet B. Walton
Purdue University

This study is focused on a statewide implementation of the core conceptual framework for effective professional development in the United States. and associated impact on teacher quality and attitudes. The framework included content focus, active learning, coherence, duration, and collective participation. There were 252 participants in the study, who were involved in one of the five science or six mathematics programs. Findings indicated a significant impact on teacher quality for all participating programs. Teacher attitudes were positive at the baseline, and remained positive with some improvement at the end of the program. However, teacher perceived preparedness increased significantly across the duration of the program. Discussion of the findings and implications for professional development programs internationally are shared.

In a global environment where governments are setting ambitious goals for educational achievement, a key challenge in educational leadership is ensuring that teachers are well prepared in terms of their content knowledge, pedagogical practices, and attention to student growth (Stewart, 2013). There is wide acknowledgment that providing ongoing learning opportunities for teachers is a critical factor in sustaining a high-quality teacher workforce and in implementing educational reforms (e.g., Cohen & Hill, 2001; Darling-Hammond, 2010; Fullan, 2010), and that educational leaders play key roles in establishing the trust relationships and workforce environments that foster teachers' willingness and ability to implement changes in their practice (Kondakci, Beycioglu, Sincar, & Ugurlu, 2017; Liu & Hallinger, 2017). Over two decades of educational effectiveness research have demonstrated the link between teacher quality and student learning (e.g., Author et al., 2007; Darling-Hammond, 2010; Muijs & Reynolds, 2001; Scheerens & Bosker, 1997), however, educational accountability policies often target improving student performance while failing to address instructional reform. Indeed, in an arena fraught with challenges, teacher change is often recognized as the most challenging aspect of educational reform (e.g., Anderson, 2002; Desimone, Porter, Garet, Yoon, & Birman, 2002; Lee & Luykx, 2006).

The difficulties inherent in achieving sustained change in teaching practices are particularly evident in science, technology, education, and mathematics (STEM) education (e.g., Gonzales et al., 2003; Southerland, 2012). Findings from the Trends in International Mathematics and Science Study (TIMSS) indicate that student performance in science and mathematics varies widely by country and even by state within the United States, and that teachers' sense of self-efficacy in content and pedagogical knowledge are related to student achievement in both mathematics and science (Martin, Mullis, Foy, & Stanco, 2012; Mullis, Martin, Foy, & Arora, 2012). In spite of this knowledge, however, evidence of poor quality science teaching in the United States continues to mount, as supported by Banilower, Smith, Pasley, and Weiss's (2006) finding that merely 14% of science lessons in the United States met high-quality standards in a national study of science teaching. Moreover, research findings indicate that U.S. teachers, particularly at the elementary level, are underprepared to teach STEM topics and that professional development programs for STEM tend to be of short duration, fail to meet individual teachers' needs, and are largely ineffective (National Research Council, 2011).

Policymakers' emphasis on student performance and new ways of framing content (e.g., 21st century skills) are frequently not accompanied by focus on or support for measures to improve instruction in spite of the evidence demonstrating that high quality, collaborative, learner-centered professional growth experiences are necessary components of improving teacher quality and classroom instruction (Antoniou & Kyriakides, 2013; Author, 2011, 2013; Desimone, 2009; Hung & Yeh, 2013; Lee & Luykx, 2006; Putnam & Borko, 1997; Van Veen, Zwart, & Meirink, 2011). Not surprisingly, educational reforms focused on modifying systems and policies without regard

to changing classroom practices have largely failed to achieve their goals (e.g., Cohen & Hill, 2001; Johnson, 2013; Olson, Desimone, Le, Floch, & Birman, 2002).

As educational policy is developed, educational leaders must be advocates for the role of instructional improvement as a necessary condition for transforming classrooms and improving learning environments in support of systemic reform. Indeed, the knowledge base around educational reform and improving teacher quality has burgeoned over the past decade (e.g., Banilower, Heck, & Weiss, 2007; Darling-Hammond, 2010; Desimone, 2009; Johnson, 2013; Johnson, Kahle, & Fargo, 2007a, 2007b; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2007; Penuel, Fishman, Yamaguchi, & Gallagher, 2007). As this evidence base grows, however, it becomes more apparent that traditional professional development formats involving short-duration workshops delivering standardized content have been ineffective drivers of systemic change in education (Garet, Porter, Desimone, Birman, & Yoon, 2001). Instead, evidence supports the idea that professional development programs should include additional contextual constructs, considering factors such as individual teachers' needs, priorities, and levels of experience (Antoniou & Kyriakides, 2013; Flint, Zisook, & Fisher, 2011). Correspondingly, the lack of such constructs in professional development design further promulgates the challenge of reforming practice (e.g., Flint et al., 2011).

To date there have been few attempts to explore the relationship between effective professional development models and achieving large-scale, systemic reform (Desimone, 2009), and no studies are available of large-scale (e.g., state or national) implementation of effective frameworks for the delivery of professional development. Furthermore, there continues to be a dearth of literature focusing on the relationship between effective professional development programs and achieving intended results in a climate of high-stakes accountability (Desimone, 2009; Johnson, 2011; Lee & Luykx, 2006).

In this article, we report on an investigation of teacher outcomes from a professional development program enacted on a statewide basis in the United States that implemented evidence-based critical features of professional development (Desimone, 2009). The study combined evidence of teacher practices and evidence of teacher attitudes, tracking changes in practices and attitudes over time. The study builds on evidence that the core components of Desimone's conceptual model for professional development should be included in impact studies due to their promise for transforming practice and promoting student achievement gains (Desimone, 2009). Although the study was enacted in the United States, Desimone's model of professional development is particularly useful on a global scale because of its focus on the context of the professional development program and its rejection of a prescriptive "one size fits all" approach to ongoing teacher learning. Because of the flexibility of the model coupled with its potential to impact teacher practice and therefore propel the change process in education, this study will inform educational leaders in a wide variety of global contexts.

Conceptual Framework

The model of professional development used to frame this study, Desimone's (2009) core conceptual framework, is based upon an examination of empirical evidence on professional development practices from literature as well as a survey of over 1,000 U.S. teachers (Desimone, 2009; Garet et al., 2001). Based upon this evidence, Desimone identified five critical features of effective professional development: content focus, active learning, coherence, duration, and collective participation. She posits a linear "path model" (Desimone, 2009, p. 185) that links the critical features of professional development with shifts in teachers' beliefs and content knowledge, changes in instructional practices, and ultimately, gains in student achievement, connections that have rarely been explored concurrently in empirical investigations.

Content focus in professional development programs means that subject matter is linked to how children access and learn new concepts and has been linked to teacher quality in numerous studies (e.g., Banilower et al., 2007; Van Driel & Berry, 2012). This conception of content focus supersedes a simple focus on learning new content, but instead grounds content learning within the teacher's role of facilitator of learning.

Active learning refers to a professional development environment that is focused on the teacher as a learner with an emphasis on learning by inquiry (e.g., Flint et al., 2011), positioning teachers as active participants in learning content and teaching strategies. As active learners, teachers assume the role of students learning through student-centered strategies such as inquiry and cooperative learning while program staff model instructional practices. In addition, teacher participants are given time to practice their new content and pedagogical knowledge and receive feedback from their teacher peers. Active learning in professional development has been linked to improved teacher outcomes in terms of classroom practice and teacher attitudes (e.g., Darling-Hammond, 1997; Johnson, 2011; Johnson & Fargo, 2010). Furthermore, as teachers use and master these student-centered strategies and implement

them in their teaching, there is evidence of subsequent growth in student learning in science and mathematics (e.g., Czerniak, Beltyukova, Struble, Haney, & Lumpe, 2005; Gonzales et al., 2003; Ruby, 2006).

Coherence refers to the alignment of the professional development program with the real-life contexts in which teachers live and practice their professions. These contexts include individual teachers' beliefs as well as local school characteristics and policies, as well as regional and/or national governmental policies. Attention to coherence is crucial to effective professional development since first-order barriers to change, those factors that stem from teachers' practice environments and over which they may have little control (Ertmer, 1999) can present substantial challenges for even teachers most desirous of implementing change. Implementation of new practices can be hampered by a variety of structural factors, including school policies and resource availability, state-mandated curriculum, and standardized testing, suggesting that professional development designers should attend to the specific contexts in which teachers practice (Johnson, Bolshakova, & Waldron, 2014). Research has demonstrated that coherence is critical to achieving fidelity of teacher implementation of learning (e.g., Buxton, Lee, & Santau, 2008; Flint et al., 2011; Fullan, 1993; Johnson, 2013; Penuel et al., 2007), a crucial link in the path to improved student outcomes.

Duration, including not only the number of contact hours, but also the period of time over which the professional development program is conducted, is the third of Desimone's (2009) critical features. In particular, Desimone posits that extended professional development contact, a minimum of 20 hours of activities delivered either over a course of months or through more intensive experiences (e.g., summer workshops) with follow-up contact, is a feature of effective professional development. Research findings support the relationship between duration of professional development and positive outcomes for students and teachers (e.g., Banilower et al., 2007; Johnson, Bolshakova, & Waldron, 2014; Johnson & Fargo, 2010; Supovitz & Turner, 2000).

Collective participation, the fifth of Desimone's (2009) critical features of effective professional development, means that a cohort of teachers from a single school or district undertake the training, allowing for support for and discourse about changes in practice over an extended period of time. Having the support of active learning communities can increase the likelihood of classroom implementation of practices (Johnson, Bolshakova, & Waldron, 2014) and has been linked to positive teacher quality outcomes in terms of their content knowledge and instructional practices (e.g., Borko, 2004; Desimone et al., 2002; Hung & Yeh, 2013; Johnson & Fargo, 2010).

Research Questions and Context

This study examined a statewide STEM teacher professional development program in the United States, and sought to answer the overarching research question, "Does participation in a statewide STEM professional development program structured around critical features of effective professional development have a positive impact on teacher outcomes?" We answered this question by investigating changes in observed classroom practice over time, changes in teacher attitudes toward reform-based teaching over time, and changes in teachers' perceptions of their preparedness to teach STEM over time. In addition, we examined the relationship between observed classroom practices, teachers' attitudes toward reform-based teaching, and teachers' perceptions of their preparedness to teach STEM both before and after participation in the professional development program to provide evidence to answer the overarching research question.

This study was set in a state in the southeastern region of the United States. The state had secured funding through the competitive Race to the Top program, a federal program targeted toward systemic school improvement across the United States. The state was awarded over $ 500 million to implement educational reform initiatives, including a statewide STEM portfolio with a purposeful emphasis on improving STEM teacher quality and student learning through an investment of over $ 16 million. A request for proposals was released by the state to postsecondary institutions across the state soliciting applications to develop and deliver STEM professional development for K–12 teachers.

Desimone's (2009) critical features of professional development were an integral component of the program, and proposals were required to address each of the five features. The request for proposals stipulated that programs must include appropriate content focus, described as activities that combine subject matter content with pedagogy and how students learn that content. Additionally, each proposal was required to designate a leadership team consisting of faculty from both teacher education and arts and sciences divisions of applicant institutions to ensure expertise within both content and pedagogy was represented. Active learning was the required context of the professional development program, and a majority of activities (at least 80% of programming) had to engage teachers in activities such as inquiry learning and cooperative learning with facilitators modeling

pedagogical content knowledge. Attention to the coherence of programs with teachers' personal and professional contexts had to be addressed through specific program activities that addressed existing teacher beliefs about reform-based instructional practices. Additionally, content was required to align with district and state reform initiatives and policies. Program duration was also stipulated, with a requirement for 80 contact hours spread across the summer months followed by academic year sessions over an 18-month period. Finally, collective participation was mandatory for each program, and teams of at least two teachers from each participant school were required.

Eleven proposals from postsecondary institutions across the state were granted funds to deliver STEM professional development in alignment with the requirements outlined above. Funded programs included two elementary science programs, five elementary mathematics programs, three secondary science programs, and one secondary mathematics program.

Methods

Data Collection and Instrumentation

Data were collected longitudinally through multiple sources across the duration of the 18-month study to assess the impact of the professional development on transforming teacher practice. Classroom observations were conducted and survey data were collected for each participating teacher ($n = 194$) both before participating in the professional development program and again, 18 months later, at the end of the program. Teachers in the study were required to submit videos of themselves delivering 45–60 minutes of science or mathematics instruction to students before and after their professional development experiences. The survey was administered through the web-based Survey Monkey platform prior to the start of the program and again at the conclusion of the 18-month period.

Teacher classroom observations instrumentation. The Local Systemic Change classroom observation protocol (LSC) is an observation tool developed by Horizon Research, Inc. (2005–2006) that can be used to assess the degree of instructional reform in mathematics and science. Using the LSC, instruction is observed and rated on a total of 32 items contained within four subscales: Design of Lesson, Implementation of Lesson, Classroom Culture, and Mathematics/Science Content. All items on each subscale are rated on a 1–5 scale with 1 meaning the indicator was "not at all" seen in the instruction, and 5 meaning the indicator was demonstrated "to a very great extent." Design of Lesson focuses on evidence of planning, use of instructional strategies, timing, and assessments used within the lesson. Implementation of Lesson assesses lesson pace, classroom management, teacher confidence, and questioning strategies implemented during the lesson. Classroom Culture addresses issues of student respect, collaboration, student interaction, and participation. Mathematics/Science Content looks at the significance and appropriateness of content for students, as well as teacher knowledge and understanding of the content being taught. Each subscale contains multiple questions that are scored individually. Table 1 shows the subscale items and the point ranges possible for each domain.

Table 1
LSC Component with Corresponding Number of Items and Possible Point Range

Component	Number of Items	Possible Point Range
Design of Lesson	10	10–50
Implementation of Lesson	7	7–35
Classroom Culture	6	6–30
Mathematics/Science Content	9	9–45

Horizon Research, Inc. provided evidence for the internal consistency for this instrument, which was determined to be high with Cronbach's alpha coefficient ranging from .92 to .97. In creating the protocol, multiple reviews were conducted by approximately 60 science and math educators to assess the content validity evidence. Multiple iterations of items were assessed to "assure broad agreement with the content of individual items and the integrity and completeness of the overall instrument" (Horizon Research, 2005, p. 1).

Teacher survey instrumentation. The Local Systemic Change Teacher Questionnaire (LSCTQ) was developed by Horizon Research, Inc. in 2003 to measure teacher attitudes toward reform-based teaching, perceptions of pedagogical preparedness, perceptions of math/science content preparedness, use of traditional teaching practices, use of practices that foster an investigative culture, use of investigative teaching practices, perceptions of principal support, and perceived impact of the professional development experience. This instrument has undergone rigorous validity and reliability investigations where expert review provided evidence that validity and internal reliability are acceptable to very good (Cronbach's alphas ranged from .47 to .94) (Germuth, Banilower, & Shinkus, 2003). Test-retest reliability was also shown to be acceptable with all correlation coefficients at statistically significant levels with ($p < .05$), ranging from .604 to .766.

For the purpose of this study, we used the attitudes toward teaching (10 items; 10–40 point range) and

pedagogical preparedness (19 items; 19–76 point range) composite scales since these aligned with our study's research questions. Attitudes toward teaching items were rated on a 4-point importance scale (Not Important, Somewhat Important, Fairly Important, Very Important), and pedagogical preparedness items were rated on a 4-point preparedness scale (Not Adequately Prepared, Some-what Prepared, Fairly Well Prepared, Very Well Prepared). All items on each scale can be viewed at Horizon Research, Inc.'s website (http://www.horizon-research.com/LSC/news/tq_test_retest.pdf).

Sample

There were 252 teacher participants in the STEM Professional Development programs (hereafter referred to as PD). Of these teachers, 194 (77.0%) submitted video observations at pre-PD and post-PD and thus constituted the sample for this study. A majority of participants were female (86.1%, $n = 167$) and White (94.2%, $n = 179$). More of the teachers in the sample participated in mathematics PD programs ($n = 121$, 62.4%) than in science PD programs ($n = 73$, 37.6%), and the sample included more elementary teachers ($n = 132$, 68.0%) than secondary teachers ($n = 62$, 32.0%).

Data Analysis

A team of raters was trained by the project director to analyze participants' classroom practices via the videos provided by teachers both before and after their professional development experiences. A set of three teacher videos (approximately 30–45 minutes in length) were first reviewed and rated by a team of evaluators using the LSC classroom observation protocol. A small sampling of videos was then viewed and scored by all raters collaboratively, and others were scored independently and compared with the goal of calibration. A primary evaluator viewed and scored all teacher videos. A second evaluator then scored a small sample across all programs for each set of videos (pre-PD and post-PD) to establish inter-rater reliability. Inter-rater reliability was at acceptable levels as reflected by the following median weighted Kappa coefficients for items within each domain on the LSC classroom observation protocol: Design of Lesson = .92, Implementation of Lesson = .88, Classroom Culture = .87, and Math/Science Content of Lesson = .91. SPSS for Windows, Release Version 17.0 (© SPSS, Inc., 2008, Chicago, IL, www.spss.com) was used for all statistical analyses in this study.

Longitudinal Impact of STEM PD

Multiple paired samples *t*-tests were conducted to assess the impact of the STEM PD over time (pre-PD to post-PD). In each paired samples *t*-test the independent variable was time (pre-PD to post-PD, or participation in PD). Observed classroom practice was the first dependent variable assessed to suggest change over time as a result of STEM PD and was measured by the following LSC classroom observation protocol subscales: Design of Lesson, Implementation of Lesson, Classroom Culture, and Mathematics/Science Content. Additionally, LSCTQ survey subsections of teacher attitudes toward reform-based teaching and pedagogical preparedness were assessed over time as indicators of STEM PD impact. Since we had hypothesized that there would be an increase in all outcome variables as a result of participating in the STEM PD programs, one-tailed tests were utilized.

Relationship Between Classroom Practice, Attitudes, and Preparedness Over Time

A multivariate Pearson correlation analysis between composite scores from all subsections of LSC classroom observation protocol and the composite scores from two LSCTQ survey sections was run to assess the relationship between observed classroom practice, teacher attitudes toward reform-based teaching, and teacher perceptions of preparedness. This analysis was conducted twice: once using pre-PD data and a second time using post-PD data. Pre-PD and post-PD relationships were then descriptively compared to look for stability of relationships over time. Since we had hypothesized that, regardless of time, teachers with higher scores for observed classroom practices would also have higher scores for attitudes toward and perceived preparedness for implementing reform-based teaching practices, one-tailed tests were conducted.

Results

Longitudinal Impact of STEM PD

Data analysis identified statistically significant positive changes in teachers' observed classroom practice on LSC classroom observation protocol subscales (Design of Lesson, Implementation of Lesson, Classroom Culture, and Mathematics/Science content) over time ($p < .001$). Effect sizes for increases in all classroom practice over time were medium to large (ranging from $\eta_p^2 = .112$ to .318). This suggests that time, or participation in PD, was responsible for 11.2–31.8% of the variance in classroom practice change. In practical terms, teachers' observed practice scores grew from an average of a solid Level 2 (Elements of Effective Instruction) at pre-PD to approaching or at a Level 3 (Beginning of Effective Instruction) at their end-of-year observations.

Significantly positive changes in teachers' attitudes ($p < .05$) and perceptions of preparedness ($p < .001$) toward

Table 2
Paired-Samples t-test Results with Means and SDs for Assessment of Longitudinal Impact of STEM PD on Teacher Observations and Survey Components

Instrument	Pre-PD	Post-PD			
Outcome Variable	*M (SD)*	*M (SD)*	*df*	*t*	η_p^2
LSC—Teacher Observations					
Design of Lesson	22.24 (5.53)	24.68 (5.82)	193	4.93*·**	.112
Implementation of Lesson	17.06 (4.45)	19.89 (4.89)	193	7.18***	.211
Classroom Culture	15.35 (3.77)	18.02 (4.36)	193	7.57***	.229
Math/Science Content	23.74 (5.11)	28.12 (5.70)	193	9.49***	.318
LSCTQ—Teacher Survey					
Attitudes Toward Reform-Based Practices	33.30 (5.78)	34.49 (6.25)	134	1.86*	.025
Preparedness for Reform-Based Practices	30.93 (5.15)	35.22 (3.62)	133	9.57***	.408

Note. All tests conducted were one-tailed.
*$p < .05$, **$p < .01$, ***$p < .001$.

reform-based teaching over time were also identified. Effect size for change in attitudes was small ($\eta_p^2 = .025$) with only 2.5% of the variance in attitude change attributed to time (or participation in PD). In practical terms, teachers responded "Fairly Important" toward reform-based teaching practices at pre-PD while responses changed slightly post-PD, averaging between "Fairly Important" and "Very Important." Effect size for perceptions of preparedness was large ($\eta_p^2 = .408$) with 40.8% of the variance in preparedness change attributed to time (or participation in PD). In practical terms, teachers averaged between "Not Adequately Prepared" and "Some-what Prepared" at pre-PD, while post-PD reports were, on average, closer to "Some-what Prepared." Table 2 shows all descriptive statistics and full statistical results for these analyses.

Relationship Between Classroom Practice, Attitudes, and Preparedness Over Time

At pre-PD assessment, there was a statistically significant positive relationship between all observed classroom practice component composite scores (Design of Lesson, Implementation of Lesson, Classroom Culture, and Mathematics/Science Content) and attitudes toward reform-based practices. These relationships, however, were considered weak ($r = .118$ to $.238$) and resulted in nominal effect sizes ($r^2 = .014$ to $.057$). Similarly, positive relationships were found between classroom practice component composite scores and preparedness for reform-based practice instruction ($r = .088$ to $.175$; $r^2 = .008$ to $.031$). The only statistically significant relationships for preparedness were in the subscales Implementation of Lesson, and Mathematics/Science Content. In practical terms, teachers with more effective classroom practices were slightly more likely to have positive attitudes toward and perceptions of preparedness for reform-based practices at the time of pre-PD assessment.

At post-PD assessment no statistically significant relationships were found between observed classroom practice components and either attitudes toward or preparedness for reform-based practices. All relationships were considered very weak, with relationships varying from slightly negative to slightly positive ($r = -.013$ to $.044$). Effect sizes were negligible ($r^2 = .000$ to $.002$). In practical terms, these results suggest that at the time of post-PD assessment there was no relationship between teachers whose observations showed more effective teaching practices and improvement in their attitudes toward or preparedness for using reform-based practices. Table 3 shows all correlational results and Table 2 depicts all descriptive statistics (means and *SD*s).

Discussion

As the research base on what constitutes effective professional development has grown, educational leaders at all levels, from local to national, have begun to implement evidence-based programs to better leverage their investments and the likelihood of realizing educational change (e.g., Author, 2013; Banilower et al., 2007; Loucks-Horsley et al., 2007; Penuel et al., 2007). It is clear that a key to transforming STEM learning environments is addressing the formidable challenge of improving teacher quality (Darling-Hammond, 2010; Muijs & Reynolds, 2001; Scheerens & Bosker, 1997; Van Veen et al., 2011). This study revealed the potential that Desimone's (2009) research-grounded core conceptual framework for professional development may have when implemented as the basis for a large-scale STEM teacher quality program. The framework's potential lies in its flexibility to support individualized, local contexts for professional development programs (e.g., Flint et al., 2011; Hung & Yeh, 2013) and its basis in the pillars of collaboration, active learning, coherence with relevant policy

Table 3
Multivariate Correlation Analysis Between Teacher Observation Components and Survey Subsections at Pre-PD and Post-PD

	Pre-PD	
	LSCTQ—Teacher Survey	
LSC—Teacher Observation	*Reform-Based Practices Attitudes* r (r^2)	*Reform-Based Practices Preparedness* r (r^2)
Design of Lesson	.182** (.033)	.088 (.008)
Implementation of Lesson	.238*** (.057)	.175** (.031)
Classroom Culture	.118* (.014)	.095 (.009)
Math/Science Content	.159** (.025)	.129* (.017)
	Post-PD	
	LSCTQ—Teacher Survey	
LSC—Teacher Observation	*Reform-Based Practices Attitudes* r (r^2)	*Reform-Based Practices Preparedness* r (r^2)
Design of Lesson	−.002 (.000)	.044 (.002)
Implementation of Lesson	−.013 (.000)	.023 (.001)
Classroom Culture	.010 (.000)	.023 (.001)
Math/Science Content	.006 (.000)	.026 (.001)

Note. All tests conducted were one-tailed. Means and *SD*s for these tests are provided in Table 3.
*$p < .05$, **$p < .01$, ***$p < .001$.

and beliefs, as well as attention to sustained duration to support the continuum of teacher growth.

The teachers who participated in STEM professional development in this study realized significant improvement in their observed teaching practices over time ($p < .001$; ranging from $\eta_p^2 = .112$ to .318). Teachers improved the design of their lessons to reflect more student-centered, inquiry-driven, investigative approaches in which students asked their own questions and devised potential solutions to real-world problems. Furthermore, participants enhanced their abilities to implement effective STEM instruction and facilitate classroom discourse while attending to the diversity of student abilities and levels of learning. Participants' STEM content knowledge also improved across the professional development program, and teachers demonstrated deeper understandings of real world grounded content after participating. Classroom culture was another area of growth for participants in the professional development programs. Teachers were observed more skillfully and regularly attending to issues of diversity and encouraging student discussion of topics through problem solving and idea generation. Overall, the participant group mean score moved from a rating of "Elements of Effective Instruction" on the LSC classroom observation protocol rubric to "Beginning Stages of Effective Instruction" across the 18-month period of the program.

Participating teachers all had positive attitudes toward implementing reform-based instruction and preparedness to teach STEM at the baseline, preprogram administration of the LSCTQ. These attitudes and perceptions remained positive at the end of the program and all teachers reportedly felt well prepared to implement STEM instruction ($\eta_p^2 = .408$) at the end of the program. The STEM professional development program in this state not only demonstrably improved teachers' abilities to plan and implement effective lessons, but also raised their confidence and their perception of their abilities to deliver STEM instruction.

Summary and Implications

For some time now, systemic educational reform has remained an elusive target in the United States and other nations. The failure of educational reform efforts to gain traction is at least partially due to the disconnect between research, policy, and governmental agencies (e.g., Author, 2013; Cohen & Hill, 2001; Darling-Hammond, 2010). Results of this U.S. statewide study indicated a relationship between the implementation of Desimone's (2009) core conceptual framework for professional development and improved teacher quality and perceptions of preparedness to deliver effective STEM instruction, suggesting a practical means to connect research with professional development design. Moreover, findings in this study reveal the potential

of Desimone's (2009) components of professional development to act as a foundation for realizing gains in teacher quality through professional development investments. The investment in teacher quality in the U.S. state in this study resulted in improved teacher outcomes and provides key evidence for educational leaders around the globe who grapple with the challenges of designing and supporting teacher professional development programs that translate into positive teacher and student outcomes (e.g., Flint et al., 2011; Olson et al., 2002).

It is important to note that although there were significant gains in teacher quality across the 18-month period (reaching a level 3 on the LSC rubric), sustained time is needed to support teachers in making the transition to highly effective teaching (levels 4–5 on the LSC rubric). In fact, previous studies have indicated that implementation of new teaching practices is a process that may take two to three years to be fully realized (e.g., Antoniou & Kyriakides, 2013; Johnson et al., 2007a, 2007b; Johnson & Fargo, 2010). This suggests that leaders of systemic reform efforts may want to consider providing ongoing support to teacher quality initiatives extending elements of the professional development program to beyond the 18-month duration used in this program.

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PROBLEMS section

Ted Eisenberg, *Section Editor*

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to eisenbt@013.net. Solutions to previously stated problems can be seen at http://www.ssma.org/publications.

Solutions to the problems stated in this issue should be posted before January 15, 2018

▪ **5463**: *Proposed by Kenneth Korbin, New York, NY*

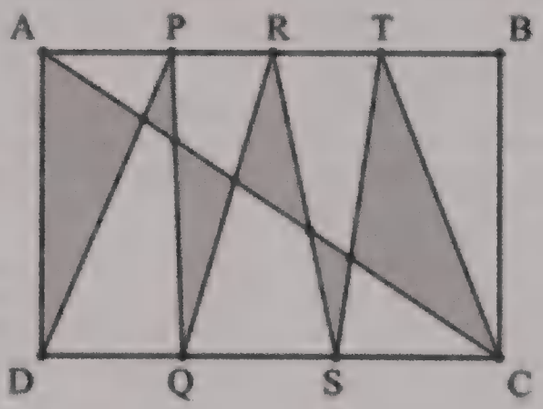
Let N be a positive integer. Find triangular numbers x and y such that $x^2+14xy+y^2=(72N^2-12N-1)^2$.

▪ **5464**: *Proposed by Ed Gray, Highland Beach, FL*

Let ABC be an equilateral triangle with side length s that is colored white on the front side and black on the back side. Its orientation is such that vertex A is at lower left, B is its apex, and C is at lower right. We take the paper at B and fold it straight down along the bisector of angle B, thus exposing part of the back side which is black. We continue to fold until the black part becomes 1/2 of the existing figure, the other half being white. The problem is to determine the position of the fold, the distance defined by x (as a function of s) which is the distance from B to the fold.

▪ **5465**: *Proposed by Arsalan Wares, Valdosta State University, Valdosta, GA*

Quadrilateral $ABCD$ is a rectangle with diagonal AC. Points P, R, T, Q, and S are on sides AB and DC, and they are connected as shown. Three of the triangles inside the rectangle are shaded pink, and three are shaded blue. Which is larger, the sum of the areas of the pink triangles or the sum of the areas of the blue triangles?



▪ **5466**: *Proposed by D.M. Bătinetu–Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "Geroge Emil Palade" School, Buzău, Romania*

Let $f:(0,+\infty)\to(0,+\infty)$ be a continuous function. Evaluate

$$\lim_{n\to\infty}\int_{\frac{n^2}{\sqrt[n]{n!}}}^{\frac{(n+1)^2}{\sqrt[n+1]{(n+1)!}}} f\left(\frac{x}{n}\right)dx.$$

▪ **5467**: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

In an arbitrary triangle ΔABC, let a, b, c denote the lengths of the sides, R its circumradius, and let h_a, h_b, h_c respectively, denote the lengths of the corresponding altitudes. Prove the inequality

$$\frac{a^2+bc}{b+c}+\frac{b^2+ca}{c+a}+\frac{c^2+ab}{a+b}\geq\frac{3abc}{2R}\sqrt[3]{\frac{1}{h_a\cdot h_b\cdot h_c}},$$

and give the conditions under which equality holds.

▪ **5468**: *Proposed by Ovidiu Furdui and Alina Sîntămărian, both at the Technical University of Cluj-Napoca, Cluj-Napoca, Romania*

Find all differentiable functions $f:\Re\to\Re$ with $f(0)=1$ such that $f'(x)=f^2(-x)f(x)$, for all $x\in\Re$.

PROBLEMS section

Ted Eisenberg, *Section Editor*

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Solutions to the problems stated in this issue should be posted before February 15, 2018

▪ **5469**: *Proposed by Kenneth Korbin, New York, NY*
Let x and y be positive integers that satisfy the equation $3x^2=7y^2+17$. Find a pair of larger integers that satisfy this equation expressed in terms of x and y.

▪ **5470**: *Proposed by Moshe Stupel, "Shaanan" Academic College of Education and Gordon Academic College of Education, and Avi Sigler, "Shaanan" Academic College of Education, Haifa, Israel*
Prove that there are an infinite number of Heronian triangles (triangles whose sides and area are natural numbers), whose side lengths are three consecutive natural numbers.

▪ **5471**: *Proposed by Arkady Alt, San Jose, CA*
For natural numbers p and n where $n \geq 3$ prove that

$$n^{\frac{1}{n^p}} > (n+p)^{\frac{1}{(n+1)(n+2)(n+3)\cdots(n+p)}}.$$

▪ **5472**: *Proposed by Francisco Perdomo and Ángel Plaza, both at Universidad Las Palmas de Gran Canaria, Spain*
Let α, β, and γ be the three angles in a non-right triangle. Prove that

$$\frac{1+\sin^2\alpha}{\cos^2\alpha}+\frac{1+\sin^2\beta}{\cos^2\beta}+\frac{1+\sin^2\gamma}{\cos^2\gamma} \geq \frac{1+\sin\alpha\sin\beta}{1-\sin\alpha\sin\beta}+\frac{1+\sin\beta\sin\gamma}{1-\sin\beta\sin\gamma}+\frac{1+\sin\gamma\sin\alpha}{1-\sin\gamma\sin\alpha}.$$

▪ **5473**: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*
Let $x_1, \cdots, x_n$ be positive real numbers. Prove that for $n \geq 2$, the following inequality holds:

$$\left(\sum_{k=1}^{n} \frac{\sin x_k}{((n-1)x_k+x_{k+1})^{1/2}}\right)\left(\sum_{k=1}^{n} \frac{\cos x_k}{((n-1)x_k+x_{k+1})^{1/2}}\right) \leq \frac{1}{2}\sum_{k=1}^{n}\frac{1}{x_k}.$$

(Here the subscripts are taken modulo n)

▪ **5474**: *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania*
Let $a, b \in \Re, b \neq 0$. Calculate

$$\lim_{n\to\infty}\begin{pmatrix} 1-\frac{a}{n^2} & \frac{b}{n} \\ \frac{b}{n} & 1+\frac{a}{n^2}. \end{pmatrix}^n.$$

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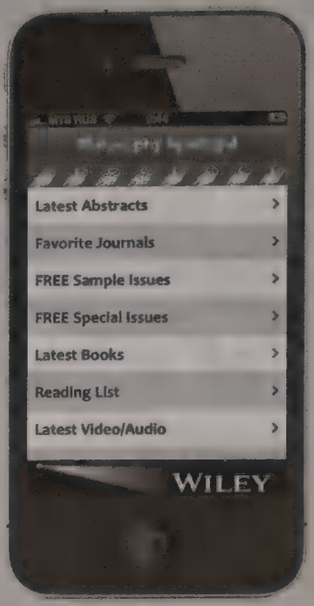

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Manuscript length, 6-26 pages including references. In addition, up to 5 pages of tables/figures/appendices will be acceptable. Tables/figures should fit on no more than one page each.

Review Information:
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No. of external reviewers: Three
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- Design/Instrumentation/Procedure
- Data Analysis/Results
- Discussion/Conclusions
- Written presentation/style
- Adds new knowledge

SCHOOL SCIENCE AND MATHEMATICS

contents

Volume 117 • Number 7-8 • November-December 2017

285 **Editorial**
The Importance of Advocacy for Integration and Effective Instruction
Carla C. Johnson

286 ***Preservice Secondary Mathematics Teachers' Conceptualizations of Equity: Access and Power as Seen Through Vignette Responses***
Brooke Max

295 ***Using Culturally Responsive Stories in Mathematics: Responses From the Target Audience***
Amy Corp

307 ***What Is a Fraction? Developing Fraction Understanding in Prospective Elementary Teachers***
Stacy Reeder, Juliana Utley

317 ***Influence of a Mathematics Teachers' Circle on Elementary Teachers' Use of Problem Solving***
Mary L. Garner, Virginia Watson, Beth Rogers, Catherine Head

329 ***Teachers' Understandings of Realistic Contexts to Capitalize on Students' Prior Knowledge***
Gloriana González

341 ***A Statewide Implementation of the Critical Features of Professional Development: Impact on Teacher Outcomes***
Carla C. Johnson, Toni Sondergeld, Janet B. Walton

350 **Problems Section**
Ted Eisenberg

352 **Problems Section**
Ted Eisenberg

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